Question 1  (18 points)
a) An electron enters the region of a uniform electric field $E = 200 \text{ N/C}$ with $v_i = 3 \times 10^6 \text{ m/s}$ at time $t = 0$. The length of the plates is $0.1 \text{ m}$. Find (i) the acceleration of the electron, (ii) the time at which it leaves the field. (iii) What is its vertical position when it leaves the field?

Solution
(i) The acceleration $a = F/m = -qE/m$

$$a = -\frac{(1.6 \times 10^{-19})(200)}{9.1 \times 10^{-31}} = -3.52 \times 10^{13} \text{ m/s}^2$$

(ii) The time to leave the field $t = \frac{x}{v_i} = \frac{0.1}{3 \times 10^6} = 3.33 \times 10^{-8} \text{ s}$

(iii) The vertical position $y = \frac{1}{2} a t^2 = \frac{1}{2} (-3.52 \times 10^{13})(3.33 \times 10^{-8})$

$$= 0.0195 \text{ m} = 1.95 \text{ cm}$$

b) A solid insulating sphere of radius $a$ carries a net positive charge $Q$ uniformly distributed throughout its volume. A conducting spherical shell of inner radius $b$ and outer radius $c$ is concentric with the solid sphere and carries a net charge $-2Q$. (i) Using Gauss’s law, find the electric field in the regions labeled 1, 2, 3, and 4. (ii) How is the charge distributed on the shell?
**Solution**

(i) Gauss Law: \[ \oint E \cos \theta \, dA = \frac{q_{\text{inside}}}{\varepsilon_o} \]

**Region 1** \[ 0 < r_1 < a \]

Choose the closed surface as a sphere of radius \( r_1 \).

\[ E (4\pi r_1^2) = \frac{Q(4/3\pi r_1^3)}{(4/3\pi a^3)\varepsilon_o} \]

\[ E = \frac{Q r_1}{4\pi a^3 \varepsilon_o} = \frac{k_e Q r_1}{a^3} \]

**Region 2** \[ a < r_2 < b \]

Choose the closed surface as a sphere of radius \( r_2 \).

\[ E (4\pi r_2^2) = \frac{Q}{\varepsilon_o} \]

\[ E = \frac{Q}{4\pi r_2^2 \varepsilon_o} = \frac{k_e Q}{r_2^2} \]

**Region 3** \[ b < r_3 < c \]

Choose the closed surface as a sphere of radius \( r_3 \).

Since the spherical shell is conducting, then \( q_{\text{inside}} = 0 \).

\[ E = 0 \]

**Region 4** \[ r_4 > c \]

Choose the closed surface as a sphere of radius \( r_4 \).

\[ E (4\pi r_4^2) = \frac{(Q - 2Q)}{\varepsilon_o} = \frac{-Q}{\varepsilon_o} \]

\[ E = \frac{-Q}{4\pi r_4^2 \varepsilon_o} = \frac{-k_e Q}{r_4^2} \]
(ii) Since the electric field in region 3 is zero and \( q_{\text{inside}} = 0 \), so the charge on the inner surface of the shell \( q_{\text{inner}} \) is

\[
q_{\text{inside}} = Q + q_{\text{inner}} = 0
\]

\[
q_{\text{inner}} = -Q
\]

The charge on the outer surface of the shell \( q_{\text{outer}} \) is

\[
q_{\text{outer}} = \left(-2Q\right) - \left(-Q\right) = -Q
\]

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**Question 2**  
*(18 points)*

a) Find an expression for the electric potential at a point \( P \) located on the central axis of a uniformly charged ring of radius \( R \) and total charge \( Q \). Use this potential to find the electric field at \( P \).

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**Solution**

The length of each ring element = \( ds = R \, d\theta \)

The charge on each ring element \( dq = \lambda \, ds = \lambda \, R \, d\theta \)

The potential due to each charge element at point \( P \):

\[
dV = \frac{dq}{4\pi\varepsilon_o \, r} = \frac{\lambda R d\theta}{4\pi\varepsilon_o \sqrt{R^2 + Z^2}}
\]

The total potential of all element at point \( P \) is by integrating:

\[
V = \int dV = \int_{\text{ring}} \frac{\lambda R d\theta}{4\pi\varepsilon_o \sqrt{R^2 + Z^2}} = \frac{\lambda R}{4\pi\varepsilon_o \sqrt{R^2 + Z^2}} \int_0^{2\pi} d\theta
\]

\[
V = \frac{\lambda R (2\pi)}{4\pi\varepsilon_o \sqrt{R^2 + Z^2}} = \frac{Q}{4\pi\varepsilon_o \sqrt{R^2 + Z^2}} \quad \text{since} \quad Q = \lambda (2\pi R)
\]
The electric field at P:

\[ E = - \frac{dV}{dZ} = - \frac{-QZ}{4\pi \varepsilon_o (R^2 + Z^2)^{3/2}} = \frac{QZ}{4\pi \varepsilon_o (R^2 + Z^2)^{3/2}} \]

b) An air-filled parallel-plate capacitor in air has a plate separation of 1.5 cm and a plate area of 25 cm\(^2\). The capacitor is charged so that it stores 0.05 J of energy. When a dielectric is inserted and completely fills the space between the plates, the energy increases to 0.125 J. Find (i) the dielectric constant of the inserted material, (ii) the change in its charge.

**Solution**

(i) Before the dielectric is inserted, the energy stored \( U_o = \frac{1}{2} C_o V^2 \)

After the dielectric is inserted, the energy stored becomes \( U = \frac{1}{2} C V^2 \)

The ratio \( U/U_o = C/C_o = k \) (The dielectric constant).

Then \( k = U/U_o = 0.125/0.05 = 2.5 \)

(ii) The capacitance before dielectric is inserted \( C_o = \varepsilon_o A/d \)

\[ C_o = (8.85\times10^{-12})(25\times10^{-4})/(1.5\times10^{-2}) = 1.475\times10^{-12} \text{ F} \]

The stored energy before inserting the dielectric \( U_o = \frac{1}{2} Q^2/C_o \)

Then \( Q = \sqrt{2U_oC_o} = \sqrt{2(0.05)(1.475\times10^{-12})} = 3.84\times10^{-7} \text{ C} \)

The stored energy after inserting the dielectric \( U = \frac{1}{2} Q^2 /C \)

Then \( Q' = \sqrt{2UC} = \sqrt{2(0.125)(2.5\times1.475\times10^{-12})} = 9.6\times10^{-7} \text{ C} \)

The increase in charge \( \Delta Q = Q' - Q = (9.6 - 3.85)\times10^{-7} = 5.75\times10^{-7} \text{ C} \)

**Question 3**  **(18 points)**

a) The magnetic force on the horizontal wire at the bottom of the circuit is 0.01 N when the magnetic field is 0.5 T. What is the total resistance of the circuit?
**Solution**

The magnetic force on the horizontal wire $F_B = I L B \sin 90$

$F_B = I L B$

The current $I = F_B/LB = (0.01)/(0.05)(0.5) = 0.4 \text{ A}$

The total resistance of the circuit $R = V/I = 24/0.4 = 60 \Omega$

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b) Derive an expression for the magnetic field inside a solenoid.

**Solution**

Ampere’s Law:

$$\int_C \vec{B} \cdot d\vec{s} = \mu_0 i$$

Consider the rectangular closed curve 1234.
Question 4  **(18 points)**

a) 40.0-mA current is carried by a uniformly wound air-core solenoid with 450 turns, a 15.0-mm diameter, and 12.0-cm length. (i) Compute the magnetic flux inside the solenoid and the energy stored in it. (ii) If the current decreases linearly at a rate of 4 mA/s, what is the induced emf?

![Diagram of a solenoid with current decreasing](image)

**Solution**

(i) The number of turns per unit length

\[ n = \frac{N}{l} = \frac{450}{12 \times 10^{-2}} = 3750 \text{ turns/m} \]

The magnetic field inside the solenoid

\[ B = \mu_0 n I = (4\pi \times 10^{-7})(3750)(40 \times 10^{-3}) = 1.885 \times 10^{-4} \text{ T} \]

The magnetic flux inside the solenoid

\[ N\Phi_B = NBA = (450)(1.885 \times 10^{-4})(2\pi \times 7.5 \times 10^{-3}) \]

\[ = 3.997 \times 10^{-3} \text{ Web} \]

The inductance

\[ L = \frac{N\Phi_B}{I} = \frac{3.997 \times 10^{-3}}{40 \times 10^{-3}} = 0.099925 \text{ H} \]
The energy stored

\[ U = \frac{1}{2} L I^2 = \frac{1}{2} (0.099925)(40 \times 10^{-3})^2 = 7.994 \times 10^{-5} \text{ J} \]

(ii) The induced emf

\[ \varepsilon = -L \frac{dI}{dt} = -(0.99925)(-4 \times 10^{-3}) = 3.997 \times 10^{-3} \text{ V} \]

b) Explain the physical meaning of each of the following Maxwell’s equations.

\[
\begin{align*}
\oint_S \vec{E} \cdot d\vec{a} &= \frac{Q_{\text{ind}}}{\varepsilon_0} \\
\oint_S \vec{B} \cdot d\vec{a} &= 0 \\
\oint_C \vec{E} \cdot d\vec{s} &= -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{a} \\
\oint_C \vec{B} \cdot d\vec{s} &= \mu_0 I_{\text{ind}} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot d\vec{a}
\end{align*}
\]

**Solution**

Equation 1 is Gauss’s Law in electrostatics

The total electric flux (which is the surface integral of the electric field) over a closed surface is equal to the total charge inside the closed surface divided by the permittivity of free space.

Equation 2 is Gauss’s Law in Magnetism

The total magnetic flux (which is the surface integral of the magnetic field) over a closed surface is equal to zero.

This indicates there is no magnetic poles.

The magnetic field lines are closed lines.

Equation 3 is Faraday’s Law of induction

The induced emf in a circuit (which is the line integral of the induced electric field around a closed path) is equal to
the negative of the rate of change of the total magnetic flux through the area bounded by the closed path.

Equation 4 Ampere - Maxwell Law

The line integral of the tangential component of the magnetic field over a closed path is equal the total current inside the closed path multiplied by the permeability of free space. The total current consists of the conduction current $i_{\text{incl}}$ and the displacement current $i_{\text{displ}}$.

The displacement current is equal to the time rate of changing magnetic flux times the permittivity of free space.

**Question 5**  
(18 points)
Choose the correct answer **justifying** your choice:

**Solution**

(1) A large charge is placed on a hollow metallic sphere that is insulated from ground. The electric field inside the sphere is ......

(a) very large  (b) very small  **(c) zero**  (d) not known

Choose a closed surface as a sphere of radius $r$ less than the metallic sphere radius.

The total charge inside the closed surface is zero.

By Guass’s Law, then $E_{\text{inside}} = 0$

(2) A uniformly charged disk of radius 35.0 cm carries charge with a density of $7\times10^{-3} \text{C/m}^2$. The electric field on the axis of the disk at 5 cm is ....

(a) $94.3\times10^3 \text{ N/C}$  (b) $8.33\times10^3 \text{ N/C}$  (c) $33.9\times10^3 \text{ N/C}$  *(d) not stated*

$$E = \sigma/2\varepsilon_o (1 - z/R) = [(7\times10^{-3})/2(8.85\times10^{-12})](1 - 5/35) = 33.9\times10^4 \text{ N/C}$$

(3) A large, flat, horizontal sheet of charge has a charge per unit area of $9.00 \mu\text{C/m}^2$. The electric field just above the middle of the sheet is ......
(a) 508.5 kN/C up  (b) 280.3 kN/C down  (c) 506.7 kN/C up  (d) not stated

\[ E = \sigma/2\varepsilon_0 = (9 \times 10^{-6})/2(8.85 \times 10^{-12}) = 508.5 \times 10^3 \text{ N/C} \]

(4) A particle with charge +q is at the origin. A particle with charge -2q is at x = 2 m on the x-axis. The electric potential is zero at x = ?

(a) 4.83 m  (b) -8.43 m  (c) -4.83 m  (d) not stated

\[ V = V_1 + V_2 = k_e q/r - 2k_e q/(2 - r) = 0 \]

q/r = 2q/(2 - r)

Solving for r

For 0 < r < 2 m

2r = 2 - r  \quad r = 0.667 m

For r < 0  \quad q/|r| = -2q/|2 - r|  \quad r = -2 m

(5) Consider a capacitor of capacitance C that is being discharged through a resistor of resistance R. After how many time constants is the charge on the capacitor reaches one-fourth its initial value?

(a) 1.39 \tau  (b) 2.4 \tau  (c) 3.54 \tau  (d) not stated

\[ q(t) = q_0 e^{-t/\tau} = \frac{1}{4} q_0 \]

\[ e^{t/\tau} = 4 \]

\[ t/\tau = \ln 4 = 1.39 \]

(6) An electron moves in a circular path perpendicular to a constant magnetic field of magnitude 1.00 mT with velocity 4x10^6 m/s. Determine the radius of the circular path.

(a) 4.56 mm  (b) 75.23 mm  (c) 22.75 mm  (d) not stated

\[ r = mv/qB = (9.1 \times 10^{-31})(4 \times 10^6)/(1.6 \times 10^{-19})(1 \times 10^{-3}) = 0.02275 \text{ m} \]

(7) A rectangular coil consists of 100 turns and has dimensions a =0.4 m and b = 0.3 m. The coil is hinged along the y-axis, and its plane makes an angle θ = 30° with the x-axis.
What is the magnitude of the torque exerted on the coil by a uniform magnetic field of 0.8 T directed in the positive x-direction when the current of 1.20 A in the direction shown is passing?

(a) 9.98 Nm  (b) 8.79 Nm  (c) 7.69 Nm  (d) not stated

\[ T = N I A B \sin \theta = (100)(1.2)(0.4 \times 0.3)(0.8) \sin 60^\circ = 9.98 \text{ Nm} \]

(8) A hollow copper tube carries a current I along its length. The magnetic field inside the tube is .................

(a) directly proportional to the current  
(b) inversely proportional to the current  
(c) zero  
(d) cannot be determined

Choose the closed path as a circle of radius r less than the tube radius.

The total current inside the closed path is zero.

By Ampere’s Law, then \( B_{\text{inside}} = 0 \)

(9) A generator produces 24.0 V when turning at 900 revolutions/min. What emf does it produce when turning at 500 revolutions/min?

(a) 26 V  (b) 24.6 V  (c) 13.3 V  (d) not stated

The emf \( \varepsilon = N B A \omega \sin \omega t \)

\[ \Omega = 2\Pi f \]

\[ \varepsilon_2/\varepsilon_1 = f_2/f_1 \]

\[ \varepsilon_2 = \varepsilon_1 f_2/f_1 = (24)(500)/(900) = 13.3 \text{ V} \]