نموذج أسئلة و اجابة مادة ميكانيكا الموائع م 1112

Benha University **College of Engineering at Banha**
Department of Mechanical Eng.
First Year Mechanical **May/18/2019**
Subject : Fluid Mechanics
Questions For Final Corrective Examination

Examiner: Dr. Mohamed Elsharnoby **Time: 180 min.**
Attempt all the following questions

Solve the following five questions, and assume any missing data

1-a) Define the system ; write down the mass, momentum and energy conservation equations and define each term in these equations.

b) Liquid enters a circular pipe of radius R with a linear velocity profile as a function of the radius with maximum velocity of Umax. After magical mixing, the velocity became uniform.

(i) Write the equation which describes the velocity at the entrance.
(ii) What is the magical averaged velocity at the exit? Assume no–slip condition.
(iii) Calculate the momentum flux correction factor for this flow.

c) Determine the magnitude and direction of the resultant force exerted on the double nozzle of Fig.1 The axes of the pipe and both nozzles lie in the horizontal plane. Both nozzle jets have a water velocity of 12m/sec.

2-a) A turbojet with a 1-m diameter inlet is being tested in a facility capable of simulating high-altitude conditions where the atmospheric pressure is 55 kPa absolute and the temperature is 267 K. The gas constant for air is 287 J/kg/K. The velocity at the inlet is 100 m/s Fig.2.. The exit diameter is 0.75 m, the exit temperature is 800 K, and the exit pressure is the local atmospheric pressure. Find the thrust produced by the turbo jet.

b) The insulated tank in Fig. 3 is to be filled from a high-pressure air supply.
Initial conditions in the tank are $T = 20°C$ and $p = 200$ kPa. When the valve is opened, the initial mass flow rate into the tank is 0.013 kg/s. Assuming an ideal gas, estimate the initial rate of temperature rise of the air in the tank.

c) The *pump-turbine* system in Fig. 4 draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower to upper reservoirs to restore the min in either direction, the friction head loss is 17 ft. Estimate the power in kW (a) extracted by the turbine and (b) delivered by the pump.

3-a) Define the following:

i) boundary layer and explain boundary layer separation ii) fully developed flow iii) entrance length L_e (write expressions for L_e in laminar and turbulent flows)

b) The velocity of viscous flow between two parallel plates is given by:

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}\left(-y^2 + yc\right) + \frac{(u_1 - u_0)}{c}y + u_0$$

Where dp/dx is the pressure gradient .u_1 and u_0 are the velocities of the upper and lower plate respectively, C the distance between the two parallel plates. For u_0 = 3m/sec, u_1 = 6m/sec, dp/dx = -20 Pa/km, $\mu = 1.849 \times 10^{-3}$ kg/ms, and C = 2m Find:

I) The flow rate per unit width and the mean velocity.
Ii) The shear stresses and friction coefficients on the lower and upper surfaces.

4-a) **Question (1):**

In **Fig.5** there are **25 m** of **5-cm** pipe, **250 m** of **2.5- cm** pipe, the pump discharge **5 L/S** of water at 20°C ($\nu = 1.005 \times 10^{-6} m^2/s$), The difference in elevation of the two ponds is **100 m**. calculate: the pump shaft horsepower (effeciency**82%**),and the gauge pressure at points **A,B**.

Assume the two pipes are smooth and use Blasisus formula for friction factor $f = 0.079 / Re^{0.25}$.

b) Assuming that the velocity profile is given by

$$\frac{u}{u_0} = f\left(\frac{y}{\delta}\right) = f(\chi)$$

where u_0 is the field velocity, δ is the boundary layer thickness and χ is y/δ.

$U/U_o = f(\chi) = \sin(\pi/2)^* \chi$ for $\chi \leq 1$ and $U/U_o = 1$ for $\chi > 1$

Find: i) δ (x) , ii) The skin friction coefficient, iii) The displacement and the momentum thicknesses, and iv) the shape factor.

5-a) The power *P* generated by a certain windmill design depends upon its diameter *D*, the air density ρ, the wind velocity *V*, the rotation rate Ω, and the number of blades *n*.

(i) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when *V* = 40 m/s and n rotating at 4800 rev/min. (ii) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude? (iii) What is the appropriate rotation rate of the prototype?

5-b) Sketch curves represent the performance and operating points of two different pumps operating singly and combined in parallel and in series.

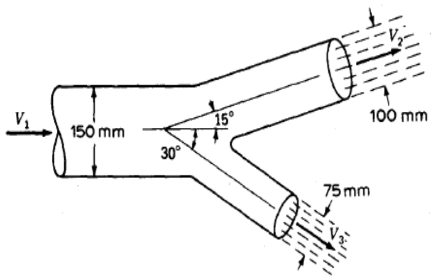

Figure 1

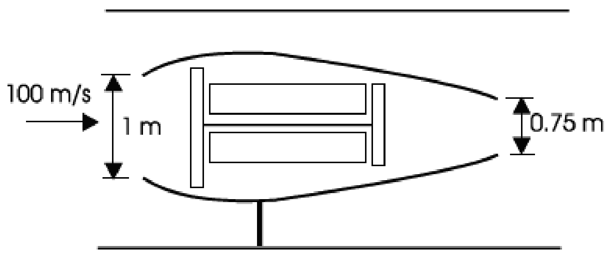

Figure 2

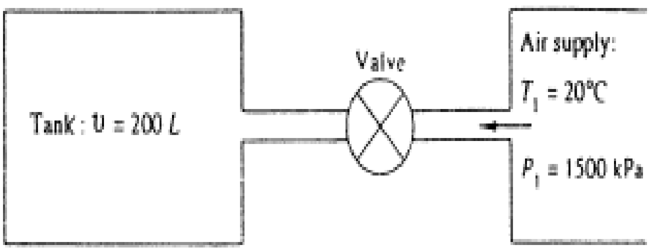

Figure 3

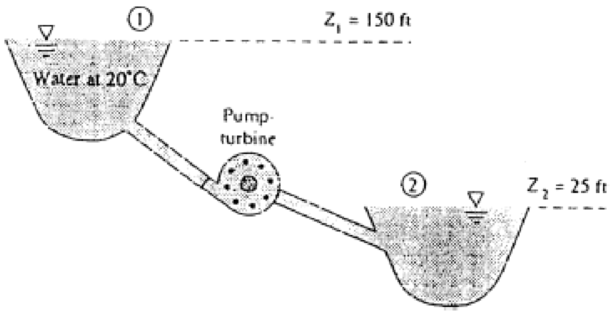

Figure 4

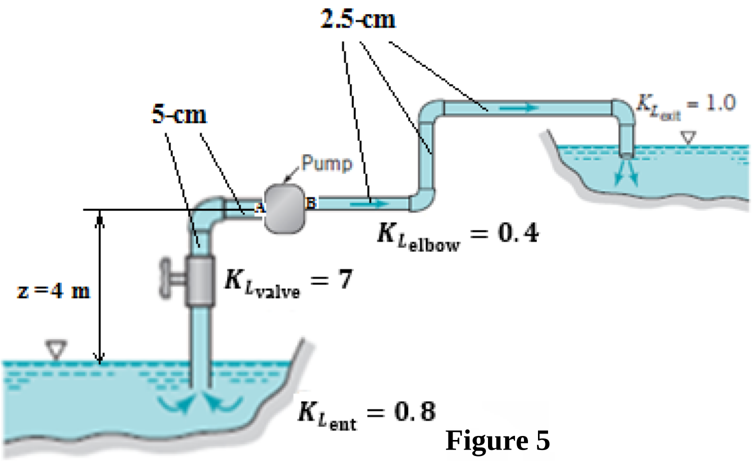

Figure 5

GOOD LUCK

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أستاذ المادة الدكتور محمد عبد اللطيف الشرنوبى
تاريخ الامتحان السبت 18 مايو 2019
الفرقة الأولى ميكانيكا

- **1-a** Recall from thermo class, that a **system** is defined as a volume of mass of fixed identity.
- **Conservation of mass** states that the mass of a system is constant.

This can be written as the following equation:

$$\frac{dm_{sys}}{dt} = 0$$

Conservation of linear momentum

which is a restatement of Newton's Second Law.

Newton's Second Law

- In equation form this is written as: $\Sigma \underline{F}_{sys} = \frac{d}{dt}(m\underline{V})_{sys}$

Where m$\underline{V}$ = the linear momentum of the system.

Conservation of Energy

- For this, use the First Law of Thermodynamics in rate form to obtain the following equation:

$$\frac{dE_{sys}}{dt} = \dot{Q}_{sys} - \dot{W}_{sys}$$

- Where E = the total energy of the system. In the above equation

$$\frac{dE_{sys}}{dt}$$

is the rate of change of system energy.

- $\dot{Q}_{sys}$ is the rate of heat added ***to*** the system $\left(\frac{\delta Q}{dt}\right)$
- $\dot{W}_{sys}$ is the rate of work done ***by*** the system $\left(\frac{\delta W}{dt}\right)$.

Because work is done by the system, the negative sign is in the equation for the first law of thermodynamics.

- Now, these conservation laws must always hold ***for a system***.

Conservation of Angular Momentum

We will have time to study this

1-b

The velocity profile is linear with radius. Additionally, later a discussion on relationship between velocity at interface to solid also referred as the (no) slip condition will be provided. This assumption is good for most cases with very few exceptions. It will be assumed that the velocity at the interface is zero. Thus, the boundary condition is

$U(r = R) = 0$ and $U(r = 0) = U_{max}$ Therefore the velocity profile is

$$U(r) = U_{max}\left(1 - \frac{r}{R}\right) \quad \textbf{(i)}$$

Where R is radius and r is the working radius (for the integration). The magical averaged velocity is obtained using the equation

. For which

$$\int_0^R U_{\max}\left(1 - \frac{r}{R}\right)2\pi r dr = U_{ave}\pi R^2$$

The integration of the equation gives

$$U_{\max}\pi\frac{R^2}{3} = U_{ave}\pi R^2 \Rightarrow U_{ave} = \frac{U_{\max}}{3} \quad \text{(ii)}$$

Calculating the momentum flux correction factor

$$\beta = \frac{1}{A}\int\left(\frac{U}{U_{ave}}\right)^2 dA = \frac{1}{\pi R^2}\int_0^R 3^2\left(1 - \frac{r}{R}\right)^2 2\pi r dr$$

$$\beta = \frac{9\times 2\pi}{\pi R^2}\int_0^R (1 - \frac{2r}{R} + \frac{r^2}{R^2})rdr$$

$$\beta = \frac{18\pi}{\pi R^2}(\frac{r^2}{2} - \frac{2r^3}{3R} + \frac{r^4}{4R^2})_0^R = \frac{18\pi}{\pi R^2}(\frac{R^2}{12}) = 1.5 \quad \text{(iii)}$$

1-c

Continuity equation

$$A_1V_1 = A_2V_2 + A_3V_3 \qquad [(\pi)(0.150)^2/4](V_1) = [(\pi)(0.100)^2/4](12) + [(\pi)(0.075)^2/4](12)$$

$$V_1 = 8.33\text{ m/s} \qquad Q_1 = A_1V_1 = [(\pi)(0.150)^2/4](8.33) = 0.147\text{ m}^3/\text{s}$$

$$Q_2 = [(\pi)(0.100)^2/4](12) = 0.094\text{ m}^3/\text{s} \qquad Q_3 = [(\pi)(0.075)^2/4](12) = 0.053\text{ m}^3/\text{s}$$

Energy equation

$$(p_1/\gamma) + [8.33^2/2(9.81)] = 0 + [12^2/2(9.81)] \qquad p_1/\gamma = 3.80\text{ m} \qquad p_1 = 37.3\text{ kPa} \qquad p_1A_1 = 0.659\text{ kN}$$

$$\sum F_x = p_1A_1 - (F_{N/L})_x = (\rho Q_2V_{2_x} + \rho Q_3V_{3_x}) - \rho Q_1V_{1_x} \qquad \rho = 10^3\text{ kg/m}^3$$

$$V_{2_x} = V_2\cos 15° = 12(0.966) = 11.6\text{ m/s} \qquad V_{3_x} = V_3\cos 30° = 12(0.866) = 10.4\text{ m/s} \qquad V_{1_x} = V_1 = 8.33\text{ m/s}$$

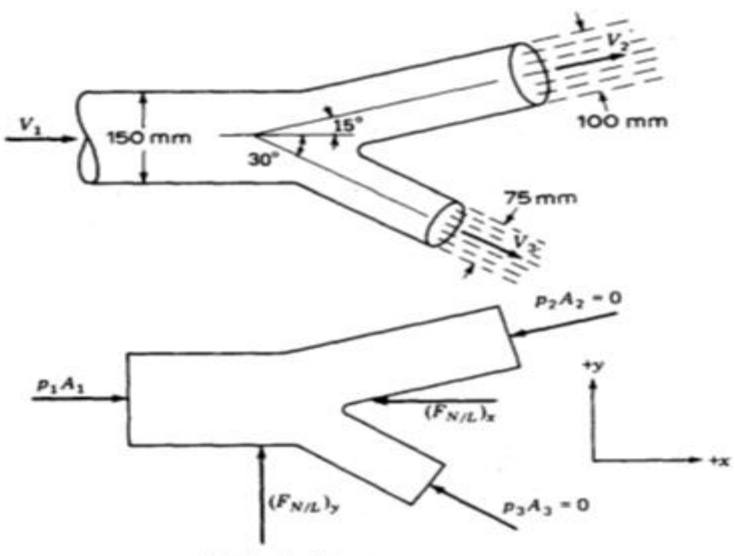
$$0.659 - (F_{N/F})_x = 10^3(0.094)(11.6) + 10^3(0.053)(10.4) - 10^3(0.147)8.33 = 0.417\text{ kN}$$

$$(F_{N/L})_x = 0.659 - 0.417 = 0.242\text{ kN}$$

$$\sum F_y = (F_{N/L})_y = (\rho Q_2V_{2_y} + \rho Q_3V_{3_y}) - \rho Q_1V_{1_y} \qquad V_{2_y} = V_2\sin 15° = 12(0.259) = 3.1\text{ m/s}$$

$$V_{3_y} = -V_3\sin 30° = -12(0.50) = -6.0\text{ m/s} \qquad V_{1_y} = 0$$

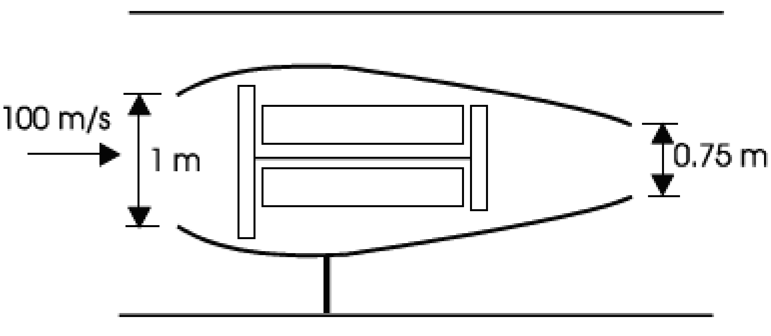
$$(F_{N/L})_y = 10^3(0.094)(3.1) + 10^3(0.053)(-6.0) - 10^3(1.47)(0) = -0.027\text{ kN}$$



Free-body diagram

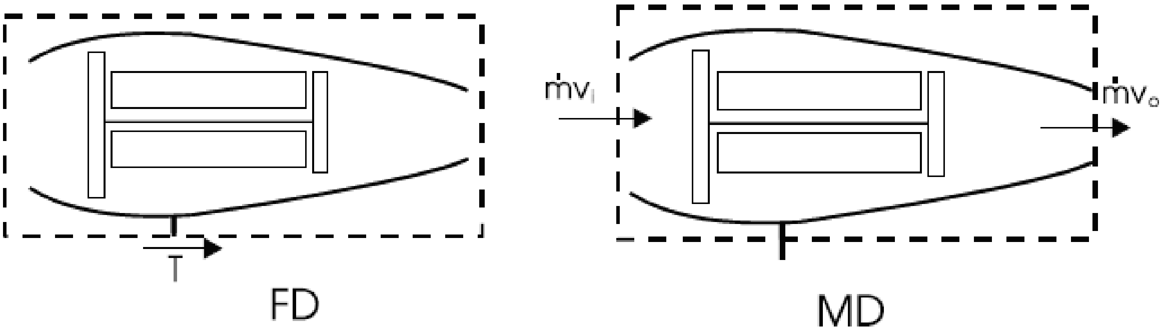
The minus sign indicates that the assumed direction of $(F_{N/L})_y$ was wrong. Therefore $(F_{N/L})_y$ acts in the negative y direction. Equal and opposite to $F_{N/L}$ is $F_{L/N}$: $(F_{L/N})_x = 0.242$ kN (in positive x direction), $(F_{L/N})_y = 0.027$ kN (in positive y direction).

2-a)



Solution

Draw the force and momentum diagrams as shown.



From the force diagram

$$\sum F_x = T$$

where T is the thrust, which is the force applied to the strut in the free-body diagram.

From the momentum diagram

$$\sum \dot{m}_o v_{o,x} - \sum \dot{m}_i v_{i,x} = \dot{m}(v_{o,x} - v_{i,x})$$

since the flow is steady and the mass flow in equals the mass flow out. Equating the force and momentum gives

$$T = \dot{m}(v_{o,x} - v_{i,x})$$

Since the control volume is stationary, the fluid velocities relative to the control volume are relative to an inertial reference frame; so

$$T = \dot{m}(V_o - V_i)$$

The density of the air at the inlet is

$$\rho_i = \frac{p_i}{RT_i} = \frac{55 \times 10^3}{287 \times 267} = 0.718 \text{ kg/m}^3$$

The mass flow is

$$\dot{m} = \rho AV = 0.718 \times \frac{\pi}{4} \times 1^2 \times 100 = 56.4 \text{ kg/s}$$

The density at the exit is

$$\rho_o = \frac{p_o}{RT_o} = \frac{55 \times 10^3}{287 \times 800} = 0.240 \text{ kg/m}^3$$

The outlet velocity is obtained from

$$V_o = \frac{\dot{m}}{\rho_o A_o} = \frac{56.4}{0.240 \times \frac{\pi}{4} \times 0.75^2} = 532 \text{ m/s}$$

The thrust is

$$T = 56.4 \times (532 - 100) = 24,360 \text{ N} = \underline{\underline{24.4 \text{ kN}}}$$

2-b) For a CV surrounding the tank, with *unsteady* flow, the energy equation is

$$\frac{d}{dt}\left(\int e\rho d\upsilon\right) - \dot{m}_{in}\left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right) = \dot{Q} - \dot{W}_{shaft} = 0, \quad \text{neglect } V^2/2 \text{ and } gz$$

$$\text{Rewrite as} \quad \frac{d}{dt}(\rho\upsilon c_v T) \approx \dot{m}_{in} c_p T_{in} = \rho\upsilon c_v \frac{dT}{dt} + c_v T\upsilon \frac{d\rho}{dt}$$

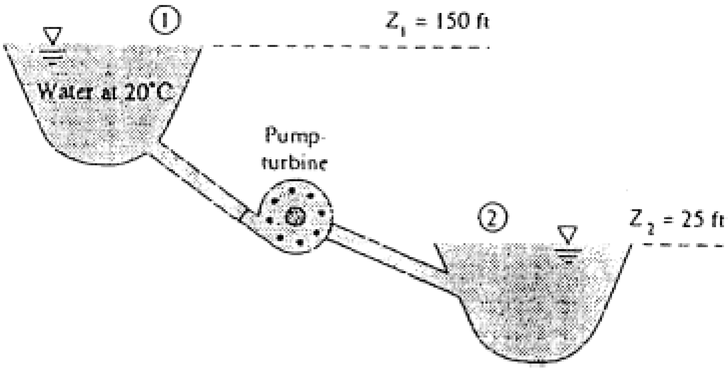
where 〉 and T are the instantaneous conditions inside the tank. The CV mass flow gives

$$\frac{d}{dt}\left(\int \rho d\upsilon\right) - \dot{m}_{in} = 0, \quad \text{or:} \quad \upsilon\frac{d\rho}{dt} = \dot{m}_{in}$$

Combine these two to eliminate ⌡(d〉/dt) and use the given data for air:

$$\left.\frac{dT}{dt}\right|_{tank} = \frac{\dot{m}(c_p - c_v)T}{\rho\upsilon c_v} = \frac{(0.013)(1005-718)(293)}{\left[\frac{200000}{287(293)}\right](0.2\ m^3)(718)} \approx \mathbf{3.2}\ \frac{°C}{s} \quad \textit{Ans.}$$

2-c



Solution: (a) With the turbine, "1" is upstream:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f + h_t,$$

$$\text{or:} \quad 0 + 0 + 150 = 0 + 0 + 25 + 17 = h_t$$

Solve for h_t = 108 ft. Convert Q = 15000 gal/min = **33.4** ft^3/s. Then the turbine power is

$$P = \gamma Q h_{turb} = (62.4)(33.4)(108) = 225{,}000\ \frac{ft\cdot lbf}{s} \approx \mathbf{410\ hp} \quad \textit{Ans. (a)}$$

(b) For pump operation, point "2" is upstream:

$$\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2=\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_f-h_p,$$

$$\text{or:}\quad 0+0+25=0+0+150+17-h_p$$

$$\text{Solve for}\quad h_p\approx 142\text{ ft}$$

The pump power is $P_{pump}=\gamma Q h_p=(62.4)(33.4)(142)=296000$ ft·lbf/s = **540 hp**. *Ans.* (b)

3-a)

i) This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the **boundary layer**.

Boundary layer is the region near a solid where the fluid motion is affected by the solid boundary.

ii) Once the boundary layer has reached the centre of the pipe the flow is said to be **fully developed**. (Note that at this point the whole of the fluid is now affected by the boundary friction.)

At a finite distance from the entrance, the boundary layers merge and the in viscid core disappears. The flow is then entirely viscous, and the axial velocity adjusts slightly further until at $x = L_e$ it no longer changes with x and is said to be fully developed, $v = v(r)$ only.

iii) The length of pipe before fully developed flow is achieved is different for the two types of flow. The length is known as the **entry length**.

The entrance length $\mathbf{L_e}$ is estimated for laminar flow to be :

$L_e/D = 0.06\ Re_D$ for laminar

$L_e/D = 4.4\ Re_D^{1/6}$ for turbulent flow

Where L_e is the entrance length; and

Re_D is the Reynolds number based on Diameter

$$u(y)=\frac{1}{2\mu}\frac{dp}{dx}\left(-y^2+yc\right)+\frac{(u_1-u_0)}{c}y+u_0$$

After putting the numerical values

$u(y)=5.4\left(-y^2+2y\right)+\frac{3}{2}y+3$,By integration

$$Q=\int_0^c\left[5.4\left(-y^2+2y\right)+\frac{3}{2}y+3\right]dy$$

$Q=16.1982 m^3/\sec$

$V_{mean}=8.0991 m/\sec$

Shear stress on the lower surface

$$\tau=\mu\frac{du}{dy}=\mu(-10.8y+12.3)$$

For lower plate y=0 $\tau_w=12.3\mu=22.7427x10^{-3}N/m^2\rightarrow C_F=0.8668x10^{-6}$

For the upper plate $\tau_w = -9.3\mu = 17.1957x10^{-5} N/m^2 \rightarrow C_F = 0.655375x10^{-6}$

4-a)For pipe 1 $v_1 = \frac{0.005}{\frac{\pi}{4}d_1^2} = 2.54648\text{m/sec}$, $R_{e1} = 126690$ f_1=0.00418737

$v_2 = \frac{0.005}{\frac{\pi}{4}d_2^2} = 10.186\text{m/sec}$ $R_{e2} = 253381$, f_2=0.00352114

$$h_{L1} = \left(f \frac{L_1}{D_1} + 0.8 + 7 + 0.4 \right) \frac{v_1^2}{2g} = 10.293685x0.3288 \quad =3.4 \text{ m}$$

$$h_{L2} = \left(f_2 \frac{L_2}{D_2} + 3x0.4 + 1 \right) \frac{v_2^2}{2g} = \quad 37.4 \ x5.261=196.82\text{m}$$

h_p=3.4 +100+196.82=300.22 m
pump power = 14.7258 kW = 20 hp
shaft horse power = 24.433 hp

4-b)

Consider f(χ) = sin χ.π/2. Then f(0) = π/2. Equation (21) assumes the form:

$$F = \int_0^1 \left(\sin.\frac{\pi}{2}\chi - \sin^2\frac{\pi}{2}\chi \right) d\chi = \int_0^1 \left(\sin\frac{\pi}{2}\chi - \frac{1}{2} + \frac{1}{2}\cos\pi\chi \right) d\chi$$

On integrating and substituting limits:

$$F = \left[-\frac{2}{\pi}\cos\frac{\pi}{2}\chi - \frac{\chi}{2} + \frac{1}{2\pi}\sin\pi\chi \right]_0^1 = \frac{2}{\pi} - \frac{1}{2} = \frac{4-\pi}{2\pi}$$

Thus we have:

$$\sqrt{2Ff'(0)} = \sqrt{\frac{4-\pi}{2\pi}} = 0.655$$

So the local skin friction coefficient is:

$$C'_F = 0.655R_x^{-\frac{1}{2}}$$

Furthermore, from equation (25), we get:

$$\frac{\delta}{x} = \sqrt{2\frac{\pi}{2}\frac{2\pi}{(4-\pi)}}.R_x^{-\frac{1}{2}} = 4.8R_x^{-\frac{1}{2}}$$

-

From equation (17), the momentum thickness can be obtained as follows:

$$\delta_2 = \delta.\int_0^\delta \frac{u}{u_0}\left(1-\frac{u}{u_0}\right)\frac{dy}{\delta}$$

or, this can be re-written as:

$$\delta_2 = \delta\int_0^1 f(\chi)\{1-f(\chi)\}d\chi$$

so that, we can say that:

$$\delta_2 = F.\delta$$

$$with$$

$$\delta_2 = \delta\frac{4-\pi}{2\pi}$$

Displacement thickness $\delta_1 = \delta\int_0^1(1-\sin\frac{\pi}{2}\chi)d\chi = \delta_1 = \delta(1-\frac{2}{\pi})$

Shape factor = $\frac{\delta_1}{\delta_2} = \frac{2(\pi-2)}{(4-\pi)} = 2.66$

5-a

i) For the function $P = \text{fcn}(D, \rho, V, \Omega, n)$ the appropriate dimensions are $\{P\} = \{ML^2T^{-3}\}$, $\{D\} = \{L\}$, $\{\rho\} = \{ML^{-3}\}$, $\{V\} = \{L/T\}$, $\{\Omega\} = \{T^{-1}\}$, and $\{n\} = \{1\}$. Using (D, ρ, V) as repeating variables, we obtain the desired dimensionless function:

$$\frac{P}{\rho D^2 V^3} = fcn\left(\frac{\Omega D}{V}, n\right) \quad \textit{Ans. (i)}$$

iii) "Geometrically similar" means that n is the same for both windmills. For "dynamic similarity," the advance ratio ($\Omega D/V$) must be the same:

$$\left(\frac{\Omega D}{V}\right)_{model} = \frac{(4800\ r/\text{min})(0.5\ m)}{(40\ m/s)} = 1.0 = \left(\frac{\Omega D}{V}\right)_{proto} = \frac{\Omega_{proto}(5\ m)}{12\ m/s},$$

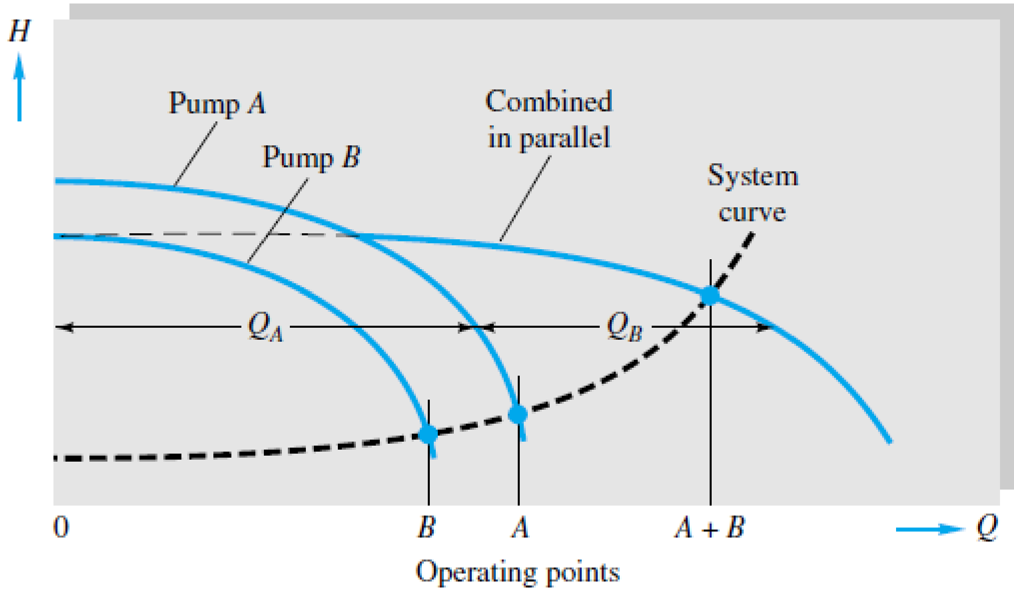
$$\text{or:} \quad \Omega_{proto} = \mathbf{144}\ \frac{\textbf{rev}}{\textbf{min}} \quad \textit{Ans.}\ \textbf{(iii)}$$

ii) At 2000 m altitude, $\rho = 1.0067\ \text{kg/m}^3$. At sea level, $\rho = 1.2255\ \text{kg/m}^3$. Since $\Omega D/V$ and n are the same, it follows that the power coefficients equal for model and prototype:

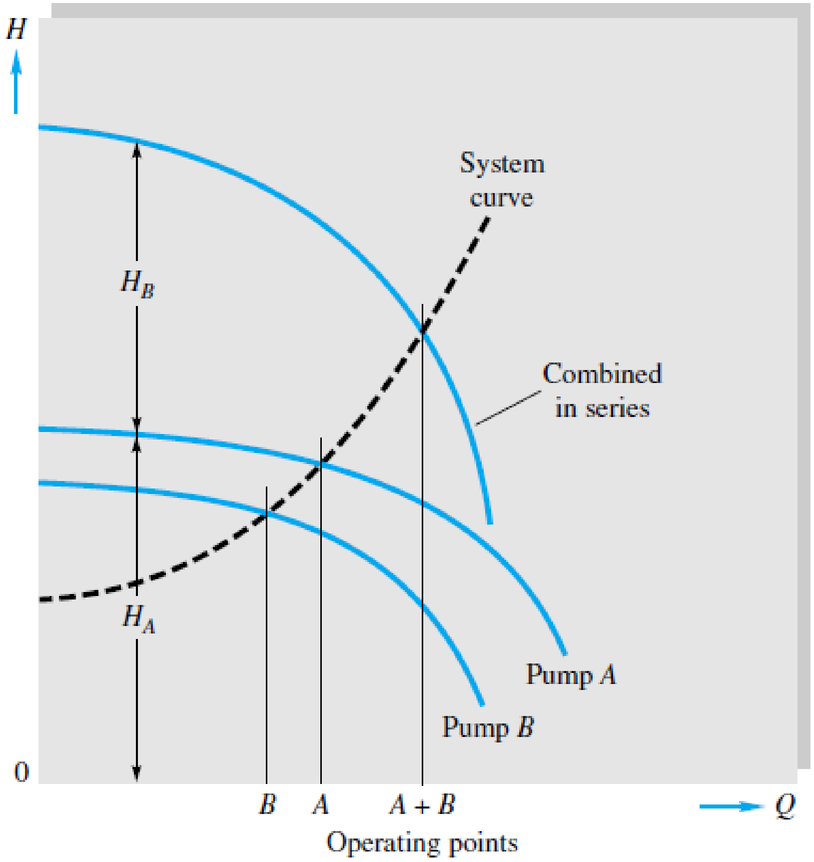
$$\frac{P}{\rho D^2 V^3} = \frac{2700 W}{(1.2255)(0.5)^2(40)^3} = \frac{P_{proto}}{(1.0067)(5)^2(12)^3},$$

$$\textit{solve} \quad \mathbf{P_{proto}} = 5990\ W \approx \mathbf{6\ kW} \quad \textit{Ans.}\ \textbf{(ii)}$$

5-b



Performance and operating points of two pumps operating singly and combined in parallel



Performance and operating points of two pumps operating singly and combined in series