

Model Answer

Question ① (10 marks)

The switch in the circuit shown in Fig.Q1. has been open for a long time. At $t = 0$ the switch is closed. Find the expression for:

- $v(t)$ when $t \geq 0^+$
- $i(t)$ when $t \geq 0$

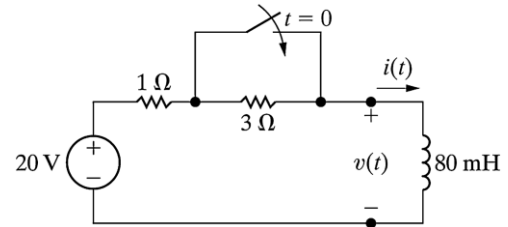


Fig.Q1

Solution

a) The switch has been open for a long time, so the initial current in the inductor is 5 A, oriented from top to bottom. Immediately after the switch closes, the current still is 5 A, and therefore the initial voltage across the inductor becomes $20 - 5(1)$, or 15 V. The final value of the inductor voltage is 0 V. With the switch closed, the time constant is $80/1$, or 80 ms. We use Eq. 7.60 to write the expression for $v(t)$:

$$\begin{aligned} v(t) &= 0 + (15 - 0)e^{-t/80 \times 10^{-3}} \\ &= 15e^{-12.5t} \text{ V, } t \geq 0^+. \end{aligned}$$

b) We have already noted that the initial value of the inductor current is 5 A. After the switch has

been closed for a long time, the inductor current reaches $20/1$, or 20 A. The circuit time constant is 80 ms, so the expression for $i(t)$ is

$$\begin{aligned} i(t) &= 20 + (5 - 20)e^{-12.5t} \\ &= (20 - 15e^{-12.5t}) \text{ A, } t \geq 0. \end{aligned}$$

We determine that the solutions for $v(t)$ and $i(t)$ agree by noting that

$$\begin{aligned} v(t) &= L \frac{di}{dt} \\ &= 80 \times 10^{-3} [15(12.5)e^{-12.5t}] \\ &= 15e^{-12.5t} \text{ V, } t \geq 0^+. \end{aligned}$$

Question ② (15 marks)

For the circuit shown in Fig.Q2, use the node voltage method to find the branch currents i_a , i_b and i_c .

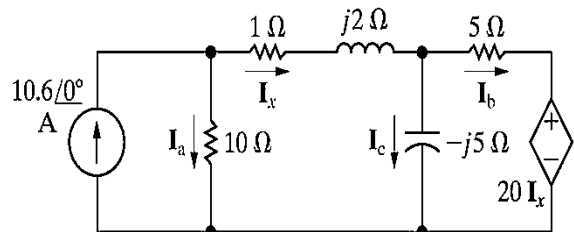


Fig.Q2

Summing the currents away from node 2 gives

$$\frac{V_2 - V_1}{1 + j2} + \frac{V_2}{-j5} + \frac{V_2 - 20I_x}{5} = 0.$$

The controlling current I_x is

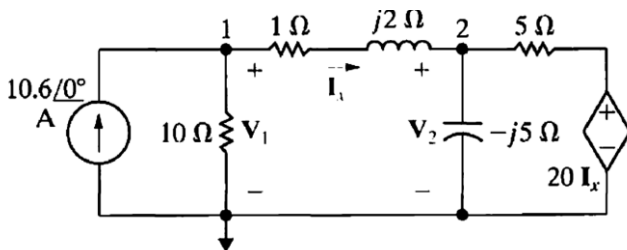
$$I_x = \frac{V_1 - V_2}{1 + j2}.$$

Substituting this expression for I_x into the node 2 equation, multiplying by $1 + j2$, and collecting coefficients of V_1 and V_2 produces the equation

$$-5V_1 + (4.8 + j0.6)V_2 = 0.$$

The solutions for V_1 and V_2 are

$$\begin{aligned} V_1 &= 68.40 - j16.80 \text{ V,} \\ V_2 &= 68 - j26 \text{ V.} \end{aligned}$$



Summing the currents away from node 1 yields

$$-10.6 + \frac{V_1}{10} + \frac{V_1 - V_2}{1 + j2} = 0.$$

Multiplying by $1 + j2$ and collecting the coefficients of V_1 and V_2 generates the expression

$$V_1(1.1 + j0.2) - V_2 = 10.6 + j21.2.$$

Hence the branch currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_1}{10} = 6.84 - j1.68 \text{ A,}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j2} = 3.76 + j1.68 \text{ A,}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_2 - 20\mathbf{I}_x}{5} = -1.44 - j11.92 \text{ A,}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_2}{-j5} = 5.2 + j13.6 \text{ A.}$$

Question 3 (15 marks)

Find the Thevenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig.Q3. Then find the maximum power that could be delivered to the impedance load connected to the terminals a,b.

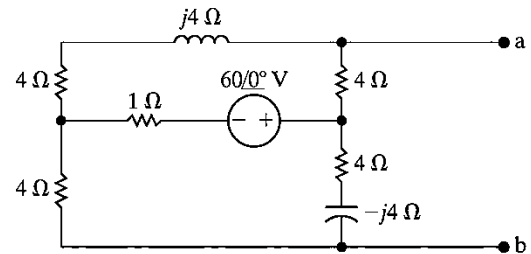
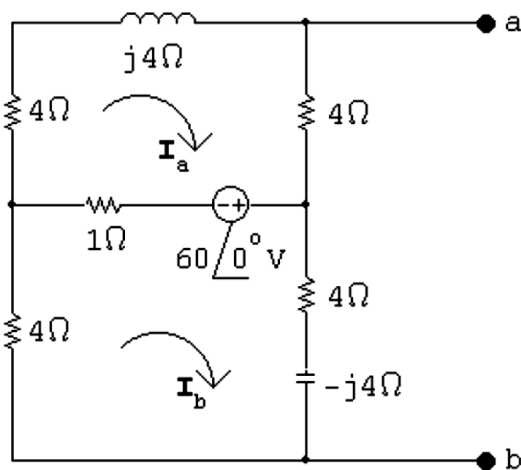


Fig.Q3

Open circuit voltage:



$$(9 + j4)\mathbf{I}_a - \mathbf{I}_b = -60/0^\circ$$

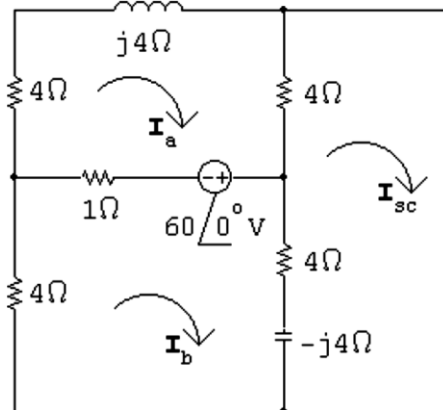
$$-\mathbf{I}_a + (9 - j4)\mathbf{I}_b = 60/0^\circ$$

Solving,

$$\mathbf{I}_a = -5 + j2.5 \text{ A; } \quad \mathbf{I}_b = 5 + j2.5 \text{ A}$$

$$\mathbf{V}_{Th} = 4\mathbf{I}_a + (4 - j4)\mathbf{I}_b = 10/0^\circ \text{ V}$$

Short circuit current:



$$(9 + j4)\mathbf{I}_a - \mathbf{I}_b - 4\mathbf{I}_{sc} = -60$$

$$-\mathbf{I}_a + (9 - j4)\mathbf{I}_b - (4 - j4)\mathbf{I}_{sc} = 60$$

$$-4\mathbf{I}_a - (4 - j4)\mathbf{I}_b + (8 - j4)\mathbf{I}_{sc} = 0$$

Solving,

$$\mathbf{I}_{sc} = 2.07/0^\circ$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_{Th}}{\mathbf{I}_{sc}} = \frac{10/0^\circ}{2.07/0^\circ} = 4.83 \Omega$$

$$P_{Lmax} = (\mathbf{V}_{Th})^2 / 4R_{Th} = 100 / (4 * 4.83) = 5.18 \text{ W}$$

Question ④ (15 marks)

Two 480 V (rms) loads are connected in parallel. The two loads draw a total average power of 40,800 W at a power factor of 0.8 lagging. One of the loads draws 20 kVA at a power factor of 0.96 leading. What is the power factor of the other load?

$$S_T = 40,800 + j30,600 \text{ VA}$$

$$S_1 = 20,000(0.96 - j0.28) = 19,200 - j5600 \text{ VA}$$

$$S_2 = S_T - S_1 = 21,600 + j36,200 = 42,154.48 / \underline{59.176^\circ} \text{ VA}$$

$$\text{rf} = \sin(59.176^\circ) = 0.8587$$

$$\text{pf} = \cos(59.176^\circ) = 0.5124 \text{ lagging}$$

Question ⑤ (15 marks)

A three-phase positive sequence Y-connected source supplies **14 kVA** with a power factor of **0.75** lagging to a parallel combination of a Y-connected load and a Δ -connected load. The Y-connected load uses **9 kVA** at a power factor of **0.6** lagging and has an a-phase current of $10 / \underline{-30^\circ}$ A.

- a) Find the complex power per phase of the Δ -connected load.
- b) Find the magnitude of the line voltage.

$$[\text{a}] S_{T\Delta} = 14,000 / \underline{41.41^\circ} - 9000 / \underline{53.13^\circ} = 5.5 / \underline{22^\circ} \text{ kVA}$$

$$S_{\Delta} = S_{T\Delta} / 3 = 1833.46 / \underline{22^\circ} \text{ VA}$$

$$[\text{b}] |\mathbf{V}_{\text{an}}| = \left| \frac{3000 / \underline{53.13^\circ}}{10 / \underline{-30^\circ}} \right| = 300 \text{ V (rms)}$$

$$|\mathbf{V}_{\text{line}}| = |\mathbf{V}_{\text{ab}}| = \sqrt{3} |\mathbf{V}_{\text{an}}| = 300\sqrt{3} = 519.62 \text{ V (rms)}$$

Question ⑥ (20 marks) 2 marks for each point.

- (1) For the circuit shown in Fig.Q6-1, At $t = (0^-)$, the circuit represents:

- a) Natural response.
- b) Step response.**
- c) None of the above.

- (2) For the circuit shown in Fig.Q6-1, If we replace the switch by a short circuit, the circuit represents an analysis type:

- a) DC Analysis.**
- b) AC Analysis.
- c) Transient Analysis.

- (3) For the circuit shown in Fig.Q6-1, If we replace the DC supply by an AC supply, the circuit represents an analysis type:

- a) DC Analysis.
- b) AC Analysis.
- c) Transient Analysis.**

- (4) If $S = 40 + j 20$, Which system has bigger value of Q_{added} , making the pf. =

- a) 0.9 lead.**
- b) 0.9 lag.

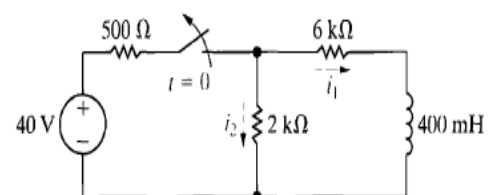


Fig. Q6-1

(5) For the circuit shown in Fig.Q6-2, The output of the circuit behaves like :

- a) **Low pass filter.**
- b) High pass filter.
- c) Band pass filter.
- d) None of the above.

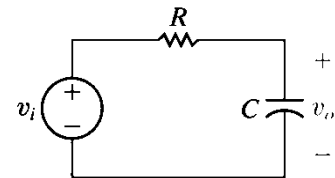


Fig. Q6-2

(6) For the circuit shown in Fig.Q6-3, the output of the circuit behaves like :

- a) Low pass filter.
- b) High pass filter.
- c) **Band pass filter.**
- d) None of the above.

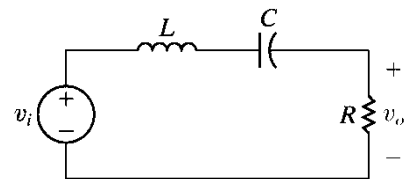


Fig. Q6-3

(7) Two factories consume the following power;

Factory (a) $\rightarrow S = 40 + j 30$

Factory (b) $\rightarrow S = 40 - j 20$

Which Factory draws higher absolute value of current from the source?

- a) Factory (a) > Factory (b).
- b) Factory (b) > Factory (a).
- c) **Factory (a) = Factory (b).**
- d) None of the above.

(8) Two loads have the following power values;

$S_1 = 40 + j 30$

$S_2 = 20 + j 20$

Find the total power if:

- a) The two loads are connected in series. **$S_T = S_1 + S_2 = 60 + j50$**
- b) The two loads are connected in parallel. **$S_T = S_1 + S_2 = 60 + j50$**

(9) In AC circuits, when the load is pure resistive. The max power transferred to the load is found by the following equation:

- a) $(V_{th})^2 / 4R_{Th}$
- b) $(V_{th})^2 / 4R_L$
- c) $(V_{th})^2 / 4Z_{Th}$
- d) $(V_{th})^2 / 4Z_L$
- e) **None of the above.**

(10) For the circuit shown in Fig.Q6-4, the value of the resistor R_o is selected to result in maximum power transfer to the 6Ω load. The max. power transferred to the 6Ω load can be calculated from the following equation. **$R_o = \text{Zero}$**

- a) $(V)^2 / 4R_L$
- b) **$(V)^2 / R_L$**
- c) $(V)^2 / 4R_o$
- d) $(V)^2 / R_o$
- e) None of the above.

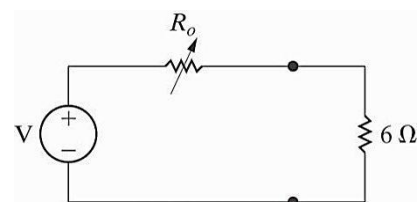


Fig. Q6-4

With best wishes

Answer All Questions

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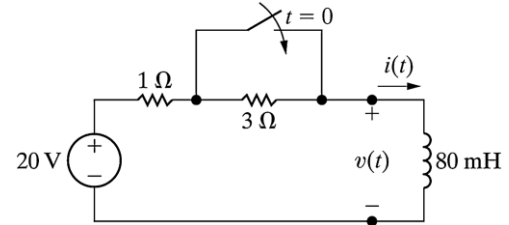


Fig.Q1

Question ② (15 marks)

For the circuit shown in Fig.Q2, use the node voltage method to find the branch currents i_a , i_b and i_c .

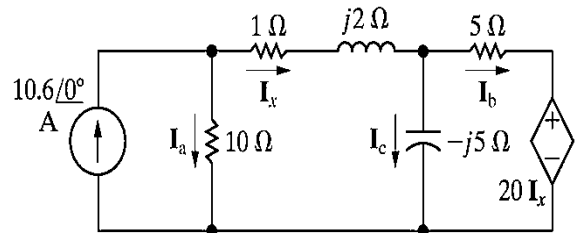


Fig.Q2

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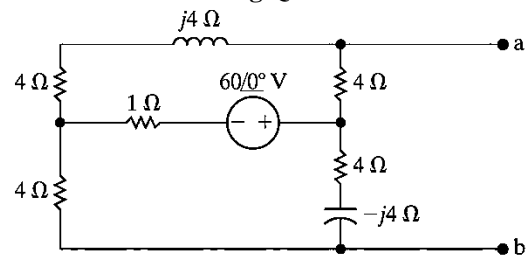


Fig.Q3

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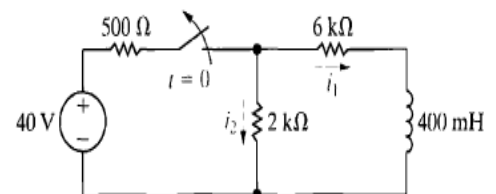


Fig. Q6-1

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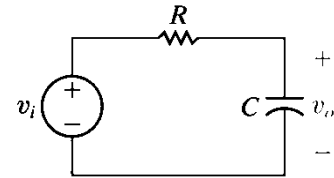


Fig. Q6-2

- (6) For the circuit shown in Fig.Q6-3, the output of the circuit behaves like :
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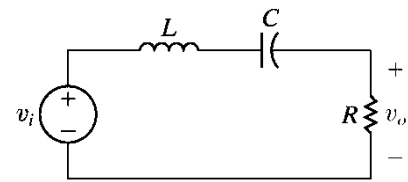


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- $(V)^2 / 4R_{\text{L}}$
- $(V)^2 / R_{\text{L}}$
- $(V)^2 / 4R_0$
- $(V)^2 / R_0$
- None of the above.

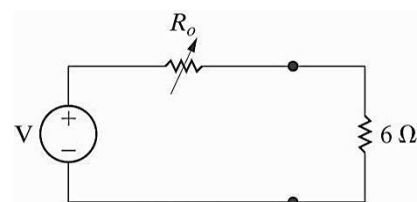


Fig. Q6-4

With best wishes