Benha University
Benha Higher Institute of Technology
Department of Mechanical Eng.
Subject: Fluid Mechanics M201

Model Answer of the Final Corrective Exam Date: Dec./28/2014

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نموذج الاجابة لمادة : ميكانيكا الموائع م ٢٠١
التاريخ الأحد ٢٨ ديسمبر ٢٠١٤
أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

Q#1 a- i) **Vapor pressure**: Pressure at which liquid boils, or pressure of vapor above liquid at equilibrium state.
Dimensions: $ML^{-1}T^{-2}$ **Units:** Pa, N/m^2, bar, psi
ii) Compressibility of the fluid: It is the volumetric strain per unit pressure change.
Dimensions: $M^{-1}LT^{2}$ **Units:** Pa^{-1}
iii) Dynamic viscosity coefficient: It measures the resistance of the fluid to flow under the effect of shear force.
Dimensions: $ML^{-1}T^{-1}$ **Units:** Kg/m.s, Pa.s, poise,..
iv) Specific weight: It is the weight per unit volume.
Dimensions: $ML^{-2}T^{-2}$ **Units:** N/m^3
v) Laminar flow: Flow at which particles move smoothly in parallel sublayers.
Turbulent flow: Flow at which particles interchange their sublayer' randomly.
No dimensions **No units**
vi) Metacenter: It is the point of intersection of axis of symmetry and line of action of buoyant force, or, point through which line of action of buoyant force is always passing.
No dimensions **No units**
vii) Boundary Layer: It is a thin layer adjacent to the solid surface in which the flow is affected by the solid surface; or It is a thin layer adjacent to the solid surface in which the shear is remarkable.
No dimensions **No units**

b)

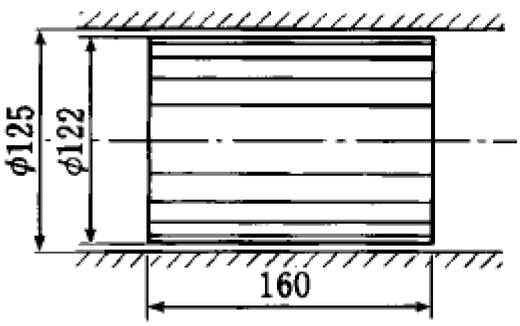


Figure 1

$\mu = \rho \nu \rightarrow \quad \nu = o.5St = 0.5cm^2/\sec$

$v = o.5\times10^{-4}\ m^2/\text{sec}, \rho = 0.9\times1000 = 900\text{kg/m}^3$

$\mu = 900\times o.5\times10^{-4} = 0.045 kg/m.s$

$$\tau = \mu\frac{du}{dy} \rightarrow \tau = 0.045\times\frac{6}{1.5\times10^{-3}} = 180Pa$$

$F = \tau A = \tau\pi DL \rightarrow F = 180\pi\times0.122\times0.16$
$F = 11.04N$

c) The compressibility of the fluid is given by: $k = \dfrac{1}{\rho}\dfrac{d\rho}{dp}$

For isentropic process $\dfrac{p}{\rho^{\gamma}} = C \rightarrow \dfrac{dp}{d\rho} = C\gamma\rho^{\gamma-1} = \dfrac{p}{\rho^{\gamma}}\gamma\rho^{\gamma-1} \rightarrow \dfrac{dp}{d\rho} = \dfrac{\gamma p}{\rho}$

$\therefore$ The isentropic compressibility $k_s = \dfrac{1}{\rho}\dfrac{\rho}{\gamma p} = \dfrac{1}{\gamma p}$

2-a)

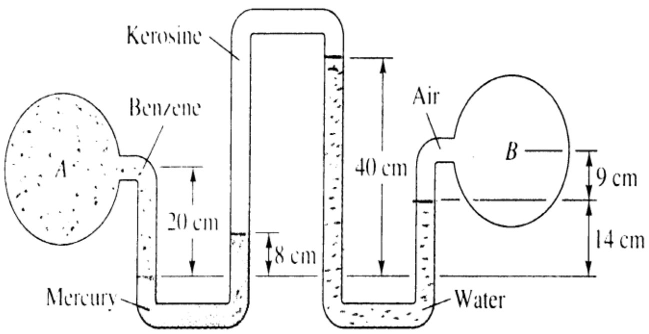
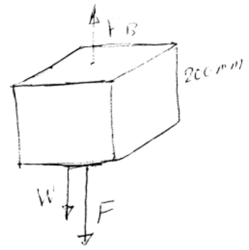


Figure 2

From figure 2 above we have:

$P_A + \rho_k g(0.2) - \rho_{Hg} g(0.08) + \rho_w g(0.4) - \rho_w g(0.14) = P_B$
$\therefore P_B = 10^5 + 804\times9.81\times0.2 - 13600\times9.81\times0.08 + 9810\times0.26$ 2-b)
$\therefore P_B = 93454.77 Pa = 0.9345477 bar$
$\therefore P_B = 13.55 psi$

2-b
The weight of the cube, (W), + the force required, (F), is equal to the buoyant force, F_B, when the cube is totally immersed in water as shown in figure 3



$$W + F = F_B \rightarrow \rho_c g V + F = \rho_w g V$$

Figure 3

F = (1000-600) g $(0.2)^3$ = 31.392 N

c) We have the equation $H = \dfrac{R^2 \omega^2}{2g}$,

Firstly we have H = 10 cm, R = 10 cm $\rightarrow\rightarrow 0.1 = \dfrac{0.1^2 \omega^2}{2g} \rightarrow \therefore$ the angular velocity $\omega = 14 rad/\sec$

For the second case H = 36 cm, R = 10 com $\rightarrow\rightarrow 0.36 = \dfrac{0.1^2 \omega^2}{2g} \rightarrow \therefore \omega = 26.577 rad/\sec$

Q#3 a) There are two cases

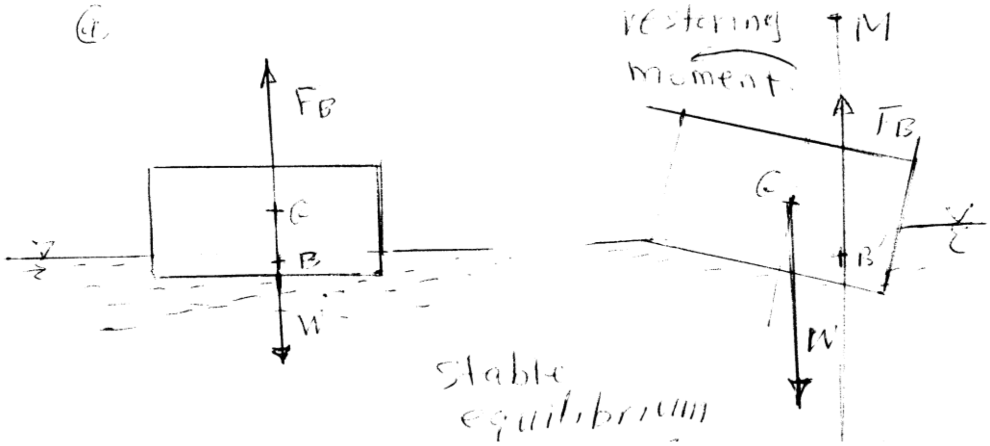


Figure 4-a

Case 1: Stable equilibrium , when the floating body is tilted to the right, a couple is created due to the deviation between the lines of actions of weight and buoyant force. This moment restore the body to its original position when the metacenter, M, is above the center of gravity, G.(figure 4-a)

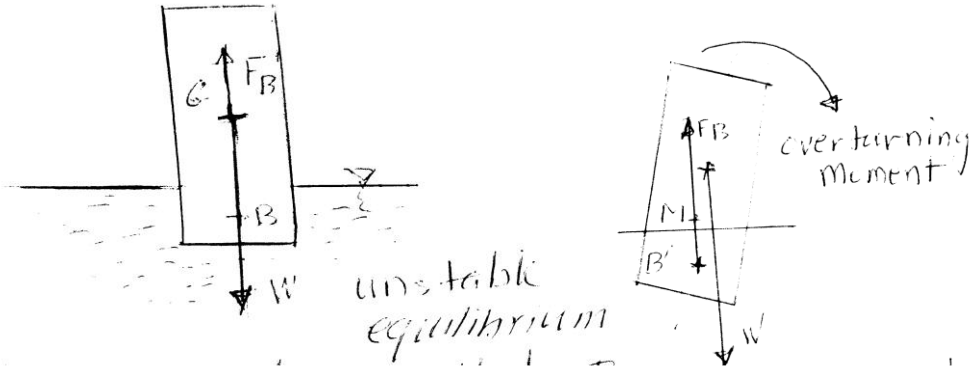


Figure 4-b

Case 2: Unstable equilibrium , when the floating body is tilted to the right, a couple is created due to the deviation between the lines of actions of weight and buoyant force. This moment turn the body over to when the metacenter, M, is below the center of gravity, G.(figure 4-b)

b-i) **Steady flow** is the flow whose properties are independent on time.
ii) **Potential flow** is the flow which has zero vorticity.
iii) **Ideal flow** is the non-viscous flow; of the flow which has zero viscosity equal zero.
iv) **Streamline** - An imaginary line in the flow that is everywhere parallel to the local velocity vectors.

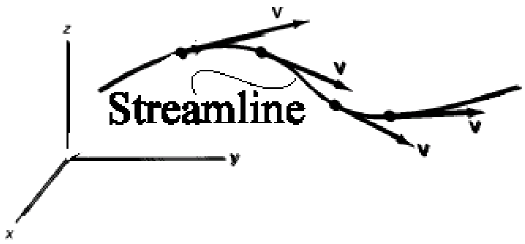


Figure 5

Streakline - An instantaneous line composed of all particles originating from a given point in the flow field; or is the locus of particles which have earlier passed through a prescribed point in space.

A timeline is a set of fluid particles that form a line segment at a given instant of time.

A pathline is the actual path traversed by a given (marked) fluid particle.

c)) i-

$$\vec{V} = (x^2 - y^2)i - 2xyj$$

$$\vec{V} = ui + vj \rightarrow\rightarrow u = x^2 - y^2, v = -2xy$$

$$\text{the acceleration } \vec{a} = a_x i + a_y j$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \rightarrow\rightarrow a_x = (x^2 - y^2)(2x) + -2xy(\text{-}2y) = 2x^3 + 2xy^2$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \rightarrow\rightarrow a_y = (x^2 - y^2)(\text{-}2y) + -2xy(\text{-}2x) = 2y^3 + 2yx^2$$

$$a_x = 80, a_y = 160 \rightarrow \vec{a} = 80i + 160j$$

ii) The flow will represent a physical flow if $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, for this flow we have $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2x - 2x = 0$

$\therefore$ The flow is a physical flow.

iii) The vorticity is given by : $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -2y + 2y = 0$

iv) $u = x^2 - y^2 = \frac{\partial \psi}{\partial y} \rightarrow \psi = x^2 y - \frac{y^3}{3} + f(x)$

$$v = -2xy = -\frac{\partial\psi}{\partial x} \rightarrow \psi = x^2 y + g(y)$$

$$\therefore \psi = x^2 y - \frac{y^3}{3}$$

The equation of the streamline passing through the point (1,2) is given by: $3x^2 y - y^3 = -2$

Q # 4

a-The assumptions made to derive Bernoulli equation:

1. Steady flow - common assumption applicable to many flows.
2. Incompressible flow - acceptable if the flow Mach number is less than 0.3.
3. Frictionless flow - very restrictive; solid walls introduce friction effects.
4. Valid for flow along a single streamline; i.e., different streamlines may have different h_o.
5. No shaft work - no pump or turbines on the streamline.
6. No transfer of heat - either added or removed.

b-

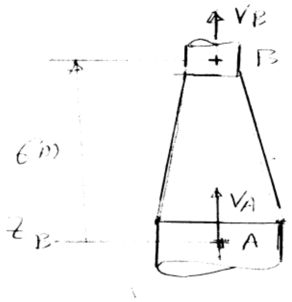


Figure 6

The continuity equation is given by: $A_A V_A = A_B V_B$

Given: $A_A = o.3\,m^2, V_A = 1.8 m/s, A_B = 0.15\,m^2$, substitute in the above equation $\rightarrow V_B = 3.6 m/s$

Applying Bernoulli equation between A and B we get: $\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$

$$P_B = P_A + \frac{\rho}{2}\left(V_A^2 - V_B^2\right) + \rho g\left(Z_A - Z_B\right)$$
$$P_B = 117000 + 500\left(1.8^2 - 3.6^2\right) + 9810(-6)$$
$$P_B = 53.28 kPa$$

c-

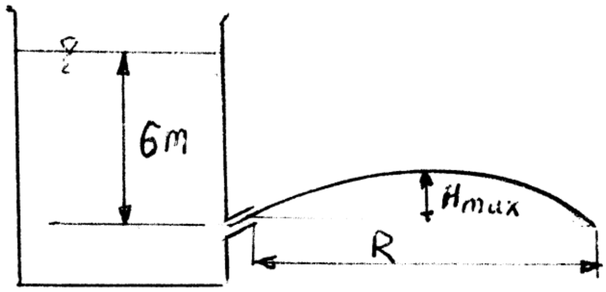


Figure 7

The maximum height H_{max} is given from the relation;

$H_{max} = H \sin^2\theta = 6.0x0.25 = 1.5$ m;

The range R is given by the equation: $R = 2H\sin 2\theta \rightarrow R = 2 \times 6 \times \sin 60^\circ$

$$R = 6\sqrt{3} = 10.392\text{m}$$

ii) The time to empty the tank is calculated as follows:

$$-A_T \frac{dh}{dt} = C_d A_o \sqrt{2gh} \rightarrow dt = -\frac{A_T}{C_d A_o \sqrt{2g}} \frac{dh}{\sqrt{h}}$$

$$t = -\frac{A_T}{C_d A_o \sqrt{2g}} \int_6^6 \frac{dh}{\sqrt{h}} = -\frac{2A_T \sqrt{h}}{C_d A_o \sqrt{2g}} \updownarrow_6^0 \rightarrow t = 680.62\text{sec}$$

Q#5

a- The flow rate Q is given from the equation:

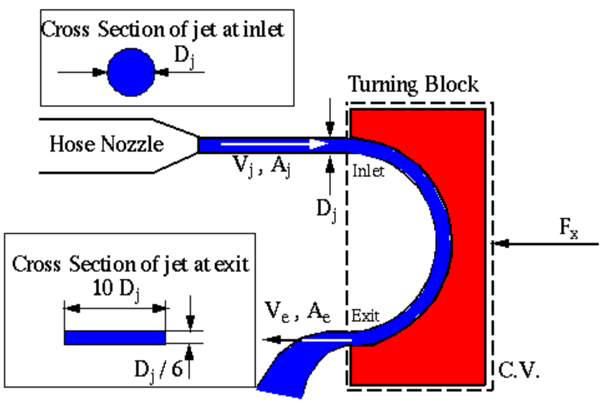


Figure 8

$$Q = AV = \frac{\pi}{4}(0.04)^2 \times 5 = 0.0062832\,\text{m}^3/\text{sec}$$

i) We apply the continuity equation to get the exit velocity Ve

$$V_e = \frac{Q}{A_e} = \frac{0.0062832}{10\times\frac{1}{6}D_J^2} = 2.3562\text{m/sec}$$

ii) $F_x = (\dot{m}V_x)_{out} - (\dot{m}V_x)_{in}$

$F_x = 6.2832(-2.3562-5) = 52.5N$

b- $\sum F_x = (\dot{m}V_x)_{out} - (\dot{m}V_x)_{in} \quad \rightarrow P_1A - P_2A - F_f = \dot{m}(\beta U_o - U_o)$

$\dot{m} = \rho Q = 39.27$ kg/sec , P1= 120 kPa, P2= 110 kPa, $\beta = \frac{4}{3}$, and U_o =5 m/s.

$\rightarrow F_f = 13.064$ N

b)

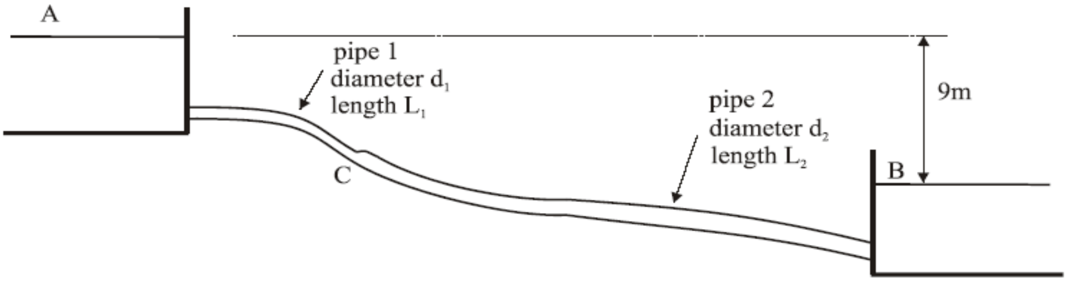


Figure 9

For the entrance use k_L = 0.5 and the exit k_L = 1.0. The join at C is sudden. For both pipes use f = 0.01.
Total head loss for the system H = height difference of reservoirs

h_{f1} = head loss for 200mm diameter section of pipe
h_{f2} = head loss for 250mm diameter section of pipe
$h_{L\ entry}$ = head loss at entry point
$h_{L\ join}$ = head loss at join of the two pipes
$h_{L\ exit}$ = head loss at exit point

Applying the energy equation between A and B

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + h_L$$

$P_B = P_A$ **= 0 (atmospheric pressure)**

$V_B = V_A$ **= 0 (large reservoirs)**

$\therefore Z_A - Z_B = H = h_L$

$h_L = h_f + h_m$**, where** h_f if the friction losses in the two pipes and h_m is the minor losses

So

$$H = h_{f1} + h_{f2} + h_{L\ entry} + h_{L\ join} + h_{L\ exit} = 9\text{m}$$

All losses are, in terms of Q:

$$h_{f1} = \frac{fL_1Q^2}{3d_1^5}$$

$$h_{f2} = \frac{fL_2Q^2}{3d_2^5}$$

$$h_{Lentry} = 0.5\frac{u_1^2}{2g} = 0.5\frac{1}{2g}\left(\frac{4Q}{\pi d_1^2}\right)^2 = 0.5 \times 0.0826\frac{Q^2}{d_1^4} = 0.0413\frac{Q^2}{d_1^4}$$

$$h_{Lexit} = 1.0\frac{u_2^2}{2g} = 1.0 \times 0.0826\frac{Q^2}{d_2^4} = 0.0826\frac{Q^2}{d_2^4}$$

$$h_{Ljoin} = \frac{(u_1 - u_2)^2}{2g} = \left(\frac{4Q}{\pi}\right)^2\frac{\left(\frac{1}{d_1^2} - \frac{1}{d_2^2}\right)^2}{2g} = 0.0826Q^2\left(\frac{1}{d_1^2} - \frac{1}{d_2^2}\right)^2$$

Substitute these into

$$h_{f1} + h_{f2} + h_{L\ entry} + h_{L\ join} + h_{L\ exit} = 9$$

and solve for Q, to give $Q = 0.158\ m^3/s$

c) We have m=6 physical parameters : power P, flow rate Q, speed of rotation of machinery N, diameter of impeller D, liquid density ρ and gh is the available energy per unit mass due to the liquid head h. We have n=3 (three basic dimensions L, M, and T).

Dimensions of the physical parameters:

$$P:[ML^2T^{-3}],$$
$$Q:[L^3T^{-1}],$$
$$gh:[L^2T^{-2}],$$
$$\rho:[ML^{-3}],$$
$$N:[T^{-1}],$$
$$D:[L]$$

We have m-n=3 π groups π_1, π_2, π_3, i.e $\Phi(\pi_1, \pi_2, \pi_3) = 0$

Choose ρ, N, D as repeating variables , $(\pi_1 = P\rho^{a_1}N^{b_1}D^{c_1}, \pi_2 = Q\rho^{a_2}N^{b_2}D^{c_2}, \pi_3 = gh\rho^{a_3}N^{b_3}D^{c_3})$

-For $\pi_1 = P\rho^{a_1}N^{b_1}D^{c_1} \rightarrow M^0L^0T^0 = [ML^2T^{-3}][ML^{-3}]^{a_1}[T^{-1}]^{b_1}[L]^{c_1}$

For M $\rightarrow 0 = 1 + a_1 \rightarrow a_1 = -1$

For L $\rightarrow 0 = 2 - 3a_1 + c_1 \rightarrow c_1 = -5$

For T $\rightarrow 0 = -3 - b_1 \rightarrow b_1 = -3 \rightarrow \pi_1 = \frac{P}{\rho N^3 D^5}$

-For $\pi_2 = Q\rho^{a_2}N^{b_2}D^{c_2} \rightarrow M^0L^0T^0 = [L^3T^{-1}][ML^{-3}]^{a_2}[T^{-1}]^{b_2}[L]^{c_2}$

For M $\rightarrow 0 = 0 + a_2 \rightarrow a_2 = 0$

For L $\rightarrow 0 = 3 - 3a_2 + c_2 \rightarrow c_2 = -3$

For T $\rightarrow 0 = -1 - b_1 \rightarrow b_1 = -1 \rightarrow \pi_2 = \frac{Q}{D^3N}$.

-For $\pi_3 = gh\rho^{a_3}N^{b_3}D^{c_3} \rightarrow M^0L^0T^0 = [L^2T^{-2}][ML^{-3}]^{a_3}[T^{-1}]^{b_3}$

For M $\rightarrow 0 = 0 + a_3 \rightarrow a_3 = 0$

For L $\rightarrow 0 = 2 - 3a_3 + c_3 \rightarrow c_3 = -2$

For T $\rightarrow 0 = -2 - b_1 \rightarrow b_1 = -2 \rightarrow \pi_3 = \frac{gh}{N^2D^2}$ so we have three non-dimensional grouping:

$\pi_1 = \frac{P}{\rho N^3 D^5}$, $\pi_2 = \frac{Q}{D^3N}$, and $\pi_3 = \frac{gh}{N^2D^2}$