Benha University

Mechanical Eng. Dept. **4th Year Mechanics**

Subject :Automatic Control **Date ٣١/١٢/2014**

Model Answer of The Final Corrective Exam

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نموذج الاجابة لمادة : التحكم الآلى م ٤٨٢

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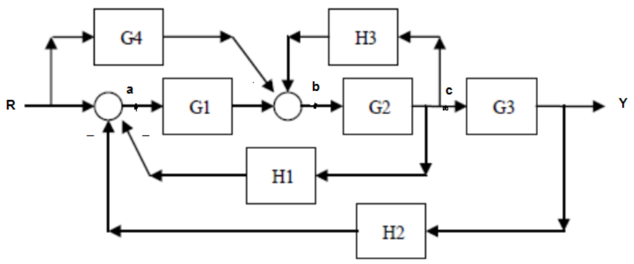
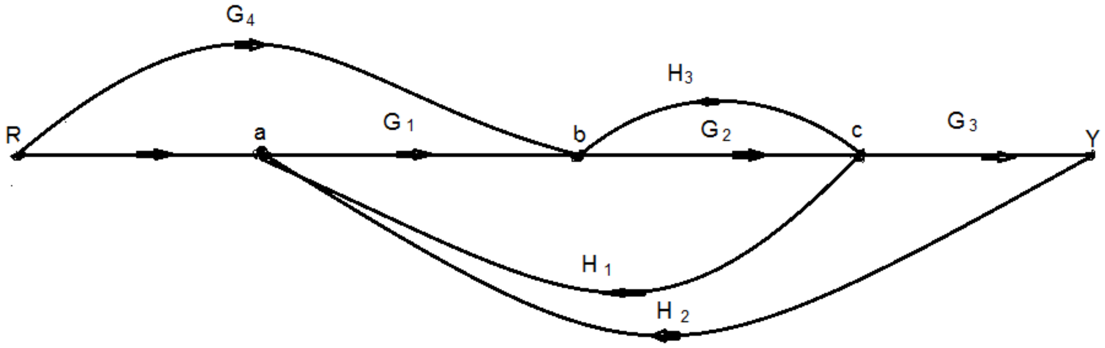
1-a)





Loops

$L_1 = -G_1G_2H_1$ $L_2 = - G_1G_2G_3H_2$ $L_3 = - G_2H_3$

Paths

$M_1 = G_1G_2G_3$ $M_2 = G_4G_2G_3$

$\Delta = 1 + G_1G_2H_1 + G_1G_2G_3H_2 + G_2H_3$

$\Delta_1 = 1$

$\Delta_2 = 1$

$TF = (M_1 \Delta_1 + M_2 \Delta_2)/ \Delta = (G_1G_2G_3+ G_4G_2G_3)/(1 + G_1G_2H_1 + G_1G_2G_3H_2 + G_2H_3)$

1-b)

FOR P=0, the characteristic equation is given by:

$S^2(S^2+2S+8) + K(S+Z) = 0$

$S^4+2S^3+8S^2+KS+KZ=0$

Construct the Huwarth array

S^4	1	8	KZ
S^3	2	K	0
S^2	(16-k)/2	KZ	0
S	$\frac{(16k-k^2-2kz)(2)}{16-k}$	0	
S^0	kz		

For stable systen

16 > K > 0, Z>0

$\frac{(16k-k^2-2kz)(2)}{16-k} > 0$

16-K-2z > 0

16 > k + 2z

ii For marginally stable system

Put $S = j\omega$

$\omega^4 - 2j\omega^3 - 8\omega^2 + jk\omega + kz = 0$

$-2\omega^3 + k\omega = 0$ $\quad k = 2\omega^2$

$\omega^4 - 8\omega^2 + kz = 0$

$\omega^4 + (2z-8)\omega^2 + = 0$

$\omega^4 + (2z-8)\omega^2 + = 0 \quad \omega = 0. \quad \omega = \pm (8-2z)^{1/2}$, $K = 16-4z$

$4 > z > 0$, $16 > k > 0$

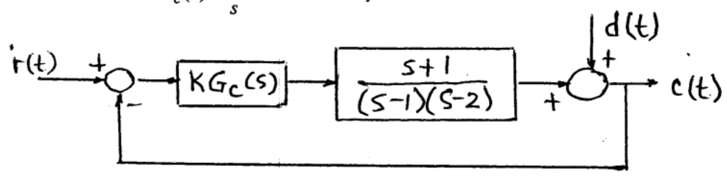
2-a)

Consider the block diagram below.

(a) With $G_c(s) = 1$, r(t)=0, and d(t)=unit step, find the range of gain K such that the steady state output due to the disturbance d(t) is 0.05d(t).

(b) Let d(t)=0. With $KG_c(s) = 18$, find the steady state step error. Also find the steady state ramp error.

(c) What effect would $G_c(s) = \frac{1}{s}$ have on the steady state errors? Note: check the root locus first.



$$\frac{C(s)}{D(s)} = \frac{1}{1 + \frac{K(s+1)}{(s-1)(s-2)}} = \frac{(s-1)(s-2)}{(s-1)(s-2) + K(s+1)} \qquad D(s) = \frac{1}{s}$$

$$\lim_{s\to 0} sC(s) = \lim_{s\to 0} \frac{(s-1)(s-2)}{(s-1)(s-2) + K(s+1)} = \frac{2}{2+K} = 0.05$$

$$\Rightarrow \quad 40 = 2 + K \Rightarrow \boxed{K = 38}$$

b) $$K_p = \lim_{s\to 0} KG_c(s)G_p(s) = \lim_{s\to 0} \frac{18(s+1)}{(s-1)(s-2)} = 9$$

$$e_{ss\,step} = \frac{1}{1+K_p} = \frac{1}{10} = \boxed{0.1 = e_{ss\,step}}$$

$e_{ss\,ramp} = \infty$ FOR A TYPE 0 CONTROL SYSTEM. TO SEE THIS,

$$K_v = \lim_{s\to 0} sKG_c(s)G_p(s) = \lim_{s\to 0} s\,\frac{18(s+1)}{(s-1)(s-2)} = 0$$

$$e_{ss\,ramp} = \frac{1}{K_v} = \infty$$

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + \frac{18(s+1)}{(s-1)(s-2)}} = \frac{(s-1)(s-2)}{(s-1)(s-2) + 18(s+1)}$$

$$e_{ss\,STEP} = \lim_{s\to 0} \cancel{s} \frac{(s-1)(s-2)}{(s-1)(s-2)+18(s+1)} \frac{1}{\cancel{s}} = \frac{2}{20} = \frac{1}{10} = 0.1.$$

$$e_{ss\,ramp} = \lim_{s\to 0} \cancel{s} \frac{(s-1)(s-2)}{(s-1)(s-2)+18(s+1)} \cdot \frac{1}{s^{\cancel{2}}} = \lim_{s\to 0} \frac{(s-1)(s-2)}{s[(s-1)(s-2)+18(s+1)]}$$

$$= \infty$$

2-b)

S^5	1	2		1
S^4	1	2		1
S^3	0		0	
	4	4		
S^2	1		1	
	0			0
s	2		0	
S^0	1			

Stable, Number of poles on the Imaginary line = 4

ii)

s^4	1	45	50
s^3	9	87	0
s^2	35.33	50	
s	74.26	0	
s^0	50		

The system is stable with all four poles on the left half plane

iii)

s^6	1	14	64	96
s^5	5	40	80	0
s^4	6	48	96	
s^3	0	0		
	2 4	96		
s^2	4	16		
s	0	0		
	8	0		
s^0	16			

The system is stable with 2 pole in the LHP and 4 on the imaginary ax

3)

a) Hand sketch the root locus of $1 + KG(s) = 0$ as K varies from 0 to $+\infty$, where

$$G(s) = \frac{s+2}{s(s+1)(s+3)^2}$$

→ open loop poles are 0, −1, −3, −3

" " zero are −2 =

Number of asymptotes are $n - m = 3$

asymptotes angle $= \frac{(2k+1)\pi}{3} = 60^\circ, 180^\circ, 300^\circ$.

Intersection of asymptotes on real axis

$$\alpha = \frac{\sum \text{Poles} - \sum \text{zeros}}{n-m} = \frac{-1, -3 - 3 + 2}{3} = \frac{-5}{3}$$

The characteristic eqn. is

$$1 + KG(s) = 0 \Rightarrow 1 + \frac{k(s+2)}{(s^2+s)(s^2+6s+9)} = 0$$

$$s^4 + 7s^3 + 15s^2 + 9s + Ks + 2K = 0$$

Construct The array.

s^4	1	15	$2K$	
s^3	7	$(9+K)$	0	
s^2	$\frac{105-9K-9}{7}$		$2K$	
s	$\frac{(96-9K)(9+K)-98K}{96-9K}$			0
1	$2K$			

$$96 - 9K > 0 \Rightarrow K < \frac{96}{9}$$

$$(96 - 9K)(9+K) - 98K > 0 \Rightarrow$$

$$864 - 81K - 98K - 9K^2 > 0$$

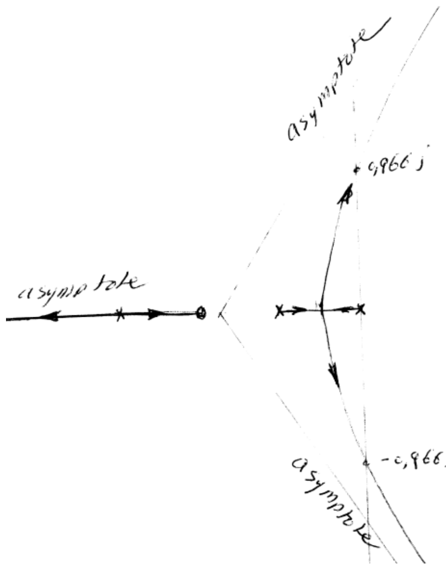
$$864 - 179K - 9K^2 > 0 \quad K \leqslant 4,$$

For $K = 4$

The Auxiliary Eqn. is $\frac{60}{7}s^2 + 8 = 0$

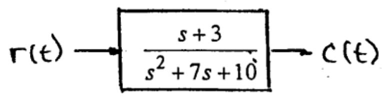
$s = \pm\sqrt{\frac{14}{15}}\,j$ points intersect The Imaginary line.

$= \pm\, 0{,}966j$



The root locus

3-b)



i) $2\zeta\omega_n = 7 \quad \omega_n^2 = 10 \Rightarrow \omega_n = \sqrt{10} = 3.16$

ii) $\zeta = \frac{7}{2\omega_n} = \frac{7}{2(3.16)} = \underline{\underline{1.107}} \Rightarrow$ OVERDAMPED SYSTEM

$\Rightarrow$ NEGATIVE, REAL, DISTINCT ROOTS.

$$s^2 + 7s + 10 = (s+5)(s+2) \Rightarrow \tau_1 = \frac{1}{5}, \ \tau_2 = \frac{1}{2}$$

$$T_s = 4\tau_{max} = 4\left(\frac{1}{2}\right) = \underline{\underline{2\ sec}}$$

iii) $C(s) = \frac{s+3}{s(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$

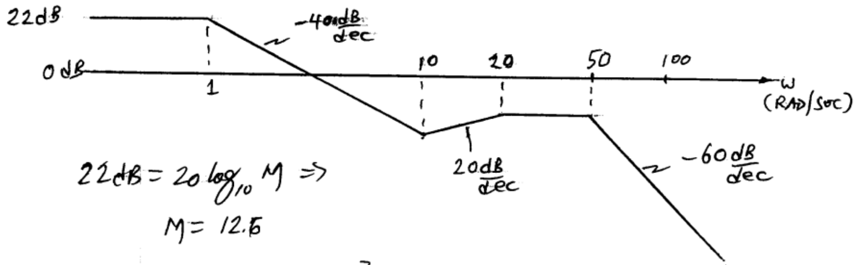
$$A = \left.\frac{s+3}{(s+2)(s+5)}\right|_{s=0} = \frac{3}{10}; \quad B = \left.\frac{s+3}{s(s+5)}\right|_{s=-2} = \frac{1}{-2(3)} = -$$

$$C = \left.\frac{s+3}{s(s+2)}\right|_{s=-5} = \frac{-2}{-5(-3)} = -\frac{2}{15}$$

$$c(t) = \frac{3}{10} - \frac{1}{6}e^{-2t} - \frac{2}{15}e^{-5t}, \quad t \geq 0$$

iv) $C_{ss} = \left|\frac{3+j2}{(5+j2)(2+j2)}\right| \cos(2t+\theta), \quad \theta = \tan^{-1}\frac{2}{3} - \tan^{-1}\frac{2}{5} - \tan^{-1}\frac{2}{2}$

4-a)



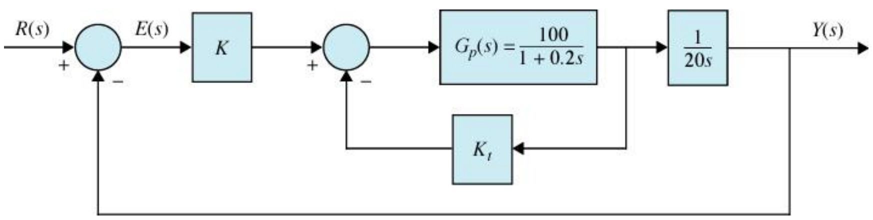
$$22dB = 20\log_{10} M \Rightarrow$$

$$M = 12.6$$

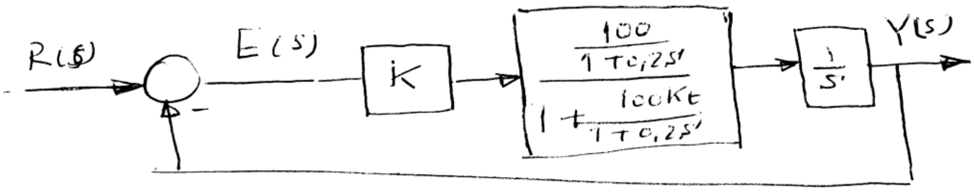
$$\frac{12.6\left(1+\frac{s}{10}\right)^3}{\left(1+\frac{s}{1}\right)^2\left(1+\frac{s}{20}\right)\left(1+\frac{s}{50}\right)^3}$$

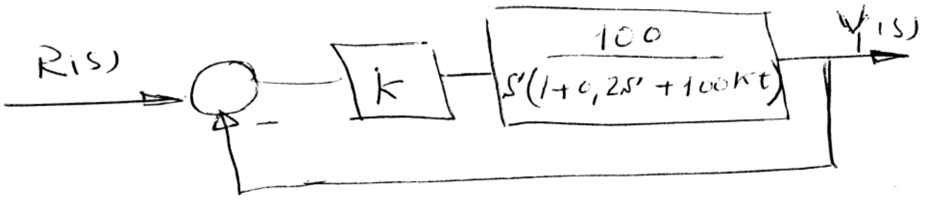
4 - b)

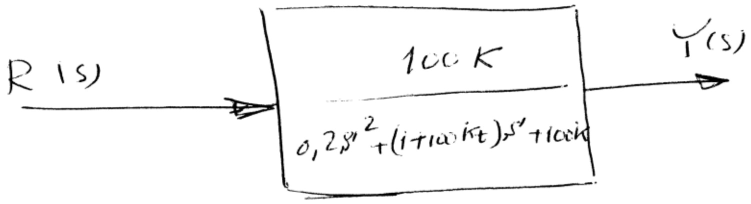
For the system given below in figure 5, estimate the values of K and K_t so that a maximum percentage overshoot of 9.6% and a settling time of 0.05 sec for a tolerance band of 1% are achieved.

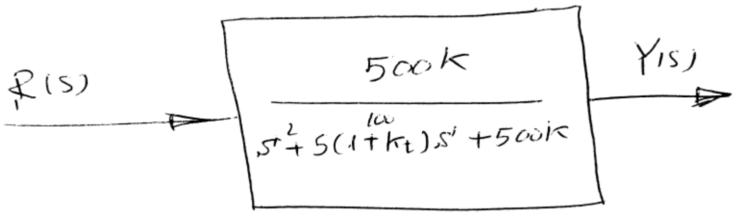


The system is reduced to









$$\omega_n^2 = 500k$$

$$2\xi\omega_n = (5 + 500k_t)$$

for 1% settling time we know

$$t_s = \frac{4.6}{\xi\omega_n} = 0{,}05$$

$$\therefore \xi\omega_n = 92$$

$$\boxed{\therefore K_t = 0{,}358}$$

$$M_P = \frac{1}{2\xi\sqrt{1-\xi^2}} = 0.096$$

$$= e^{-\pi\xi/\sqrt{1-\xi^2}} = 0{,}096$$

$$\xi = 0{,}5979 \Longrightarrow \omega_n = 153.87$$

$$\boxed{K = \frac{\omega_n^2}{500} = 47.35}$$

c) Consider the standard feedback system shown below in figure 6.

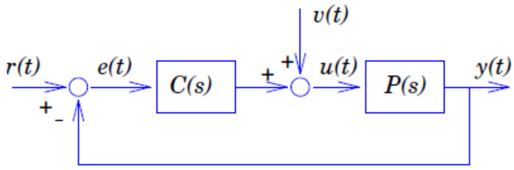
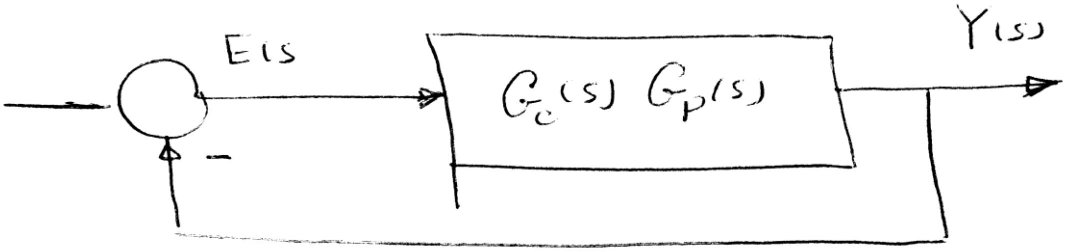


Figure 6



The characteristic Eqn. is given by:

$$1 + G_c(s)\, G_p(s) = 0$$

$$1 + \frac{K_p s + K_i}{s} \cdot \frac{1}{(s^2 + 5s)} = 0$$

$$s^3 + 5s^2 + K_p s + K_i = 0$$

$s = -4$ satisfies the eqn.

$$-64 + 80 - 4K_p + K_i = 0$$

$$16 - 4K_p + K_i = 0$$

$$K_i = 4K_p - 16$$

$$s^3 + 5s^2 + K_p s + 4K_p - 16 = 0$$

$$K_p > 4$$

$$(s^3 + 4s^2) + (s^2 - 16) + K_p(s+4) = 0$$

$$s^2(s+4) + (s+4)(s-4) + K_p(s+4) = 0$$

$$(s+4)\left[s^2 + s + (K_p - 4)\right] = 0$$

$$r_{2,3} = \frac{-1 \pm \sqrt{1 + 16 - 4K_p}}{2}$$

$$-\frac{1}{2} \pm j\sqrt{K_p - \frac{17}{4}} = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$$

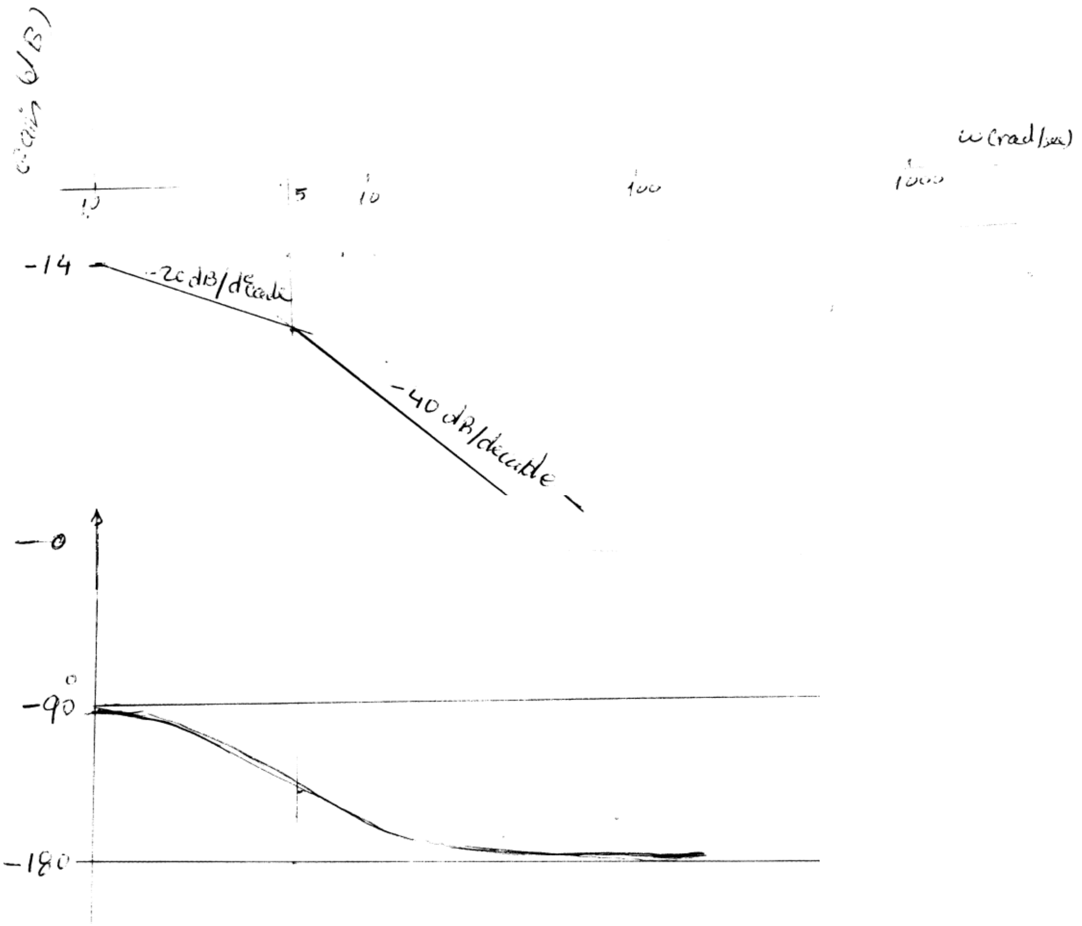
choose $\omega_n = 2 \longrightarrow \xi = 0.25$

$$4(1-\xi^2) = K_p - \frac{17}{4} \Rightarrow \frac{15}{4} + \frac{17}{4} = K_p \Rightarrow$$

$$\boxed{K_p = 8}$$

The transfer function $\frac{1}{s'(s+5)}$

$$= \frac{1}{5}\frac{1}{(s')}\frac{1}{(0,2s+1)}$$



The required B0de diagram