



Model Answer of the Final Exam

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نموذج الاجابة لمادة : ديناميكا الموائع والغازات ( م ٤٠١ )

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1-a1-a) .The equation relating the relative change in area with the relative change in velocity is given by:

$$\frac{dA}{A} = (M^2 - 1) \frac{dV}{V}$$

which may alternatively be written as:

$$\frac{dA}{dV} = (M^2 - 1) \frac{A}{V}$$

Because  $A$  and  $V$  are positive, it may be concluded from the above two equations that:

1. If  $M < 1$ , i.e., if the flow is subsonic, then  $dA$  has the opposite sign to  $dV$ , i.e., decreasing the area increases the velocity and vice versa.
2. If  $M > 1$ , i.e., if the flow is supersonic, then  $dA$  has the same sign as  $dV$ , i.e., decreasing the area decreases the velocity and vice versa.
3. If  $M = 1$  then  $dA/dV = 0$  and  $A$  reaches an extremum. From (1) and (2) it follows that when  $M = 1$ ,  $A$  must be a minimum.

b)

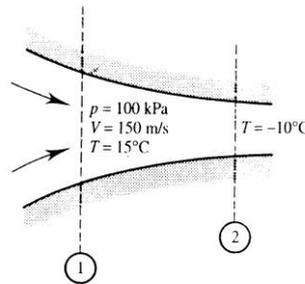


Fig.1

The absolute temperature at section 1 is  $T_1 = 273 + 15 = 288$  K

The Mach number at section 1 is  $M_1 = 0.4409$

The absolute temperature at section 2 is  $T_2 = 273 - 10 = 263$  K

From the energy equation

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2} \Rightarrow V_2 = 218$$

The Mach number at section 2 is  $M_2 = 0.671$

$$\rho_1 = \frac{p_1}{RT_1} = 1.209 \text{ kg/m}^3 \text{ from the equation } \frac{\rho_2}{\rho_1} = \left( \frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}} = 0.7969$$

$$\rightarrow \therefore \rho_2 = 0.9635 \text{ kg/m}^3$$

From the continuity equation  $\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \rightarrow \frac{A_2}{A_1} = 0.8634$

c)  $\rho_1 = \frac{P_1}{RT_1} = 0.4598$  ,  $V_1 = 872.5$  m/s

the mass flow rate  $\dot{m} = \rho_1 v_1 A_1 = 0.9628$  kg/sec.

For  $M_1 = 2.5$

$M_1 = 2.5 \rightarrow \frac{A_1}{A_1^*} = 2.6367$  ,  $\frac{P_1}{P_{o1}} = 0.0585$  to obtain the Mach number before the shock  $M_{2n1}$  we get

$\frac{A_2}{A_1^*} = \frac{A_2}{A_1} \frac{A_1}{A_1^*} = 1.978 \rightarrow$  from table  $M_{2n1} = 2.185 \rightarrow$  from the normal shock table we get the Mach

number after the shock  $M_{2n2} = 0.549$  , also we get  $\frac{P_{o2}}{P_{o1}} = 0.635$  , we have  $P_{o1} = 683,76$  kPa

$\rightarrow \therefore P_{o2} = 414.2$  kPa

From table A1 for  $M_{2n2} = 0.549$  we get  $\frac{A_2}{A_2^*} = 1.257$

$\frac{A_3}{A_2^*} = \frac{A_3}{A_2} \frac{A_2}{A_2^*} = 2.235$

The Mach number at section 3 is  $M_3 = 0.25$

The stagnation pressure at section 3 is  $P_{o3} = 414.2$  kPa

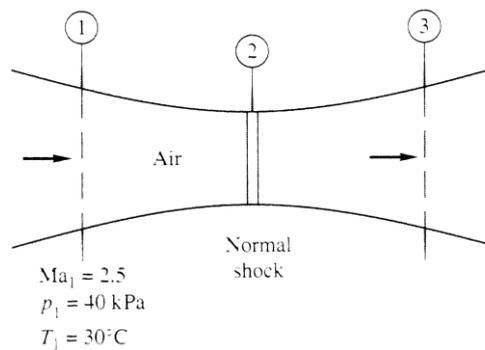


Fig.2

**2-a)** Rayleigh flow is one-dimensional frictionless flow in constant area duct with heat addition and removal

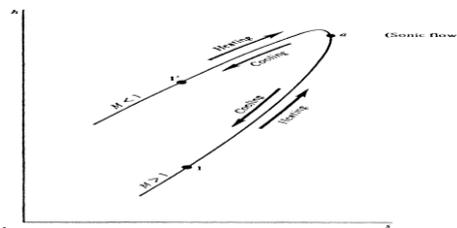


Figure3. Rayleigh curve

Heating always pushes the flow towards the sonic conditions and vice versa. This is clear on the *Rayleigh curve* shown above In detail we can conclude the following:

1. For *supersonic flow* in region 1, i.e.,  $M_1 > 1$ , when heat is added
  - a. Mach number decreases,  $M_2 < M_1$
  - b. Pressure increases,  $p_2 > p_1$
  - c. Temperature increases,  $T_2 > T_1$
  - d. Total temperature increases,  $T_{o2} > T_{o1}$
  - e. Total pressure decreases,  $p_{o2} < p_{o1}$
  - f. Velocity decreases,  $u_2 < u_1$
2. For *subsonic flow* in region 1, i.e.,  $M_1 < 1$ , when heat is added
  - a. Mach number increases,  $M_2 > M_1$
  - b. Pressure decreases,  $p_2 < p_1$
  - c. Temperature increases for  $M_1 < \gamma^{-1/2}$  and decreases for  $M_1 > \gamma^{-1/2}$
  - d. Total temperature increases,  $T_{o2} > T_{o1}$
  - e. Total pressure decreases,  $p_{o2} < p_{o1}$
  - f. Velocity increases,  $u_2 > u_1$

For heat extraction (cooling of the flow), all of the above trends are opposite.

From the development here, it is important to note that heat addition always drives the Mach numbers toward 1, decelerating a supersonic flow and accelerating

2-b) The situation under consideration is shown in Fig.4

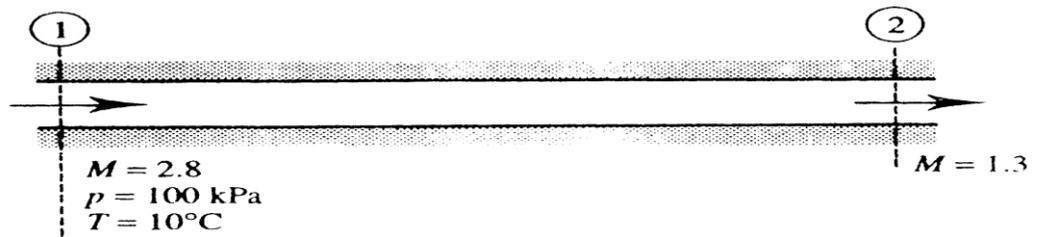


Fig.4

Now for Mach number 2.8 using the relation.

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + 0.2M^2$$

or using the software or isentropic tables gives  $T_{01}/T_1 = 2.568$ . Hence:

$$T_{01} = 2.568 \times (273 + 10) = 726.7 \text{ K}$$

Next, using the relations given above or the tables or the software for frictionless flow in a constant area duct with heat exchange gives:

For  $M = 2.8$ :

$$\frac{p}{p^*} = 0.2004, \quad \frac{T}{T^*} = 0.3149, \quad \frac{T_0}{T_0^*} = 0.6738$$

For  $M = 1.3$ :

$$\frac{p}{p^*} = 0.7130, \quad \frac{T}{T^*} = 0.8592, \quad \frac{T_0}{T_0^*} = 0.9580$$

Using these values gives:

$$p_2 = \frac{p_2/p^*}{p_1/p^*} p_1 = \frac{0.7130}{0.2004} \times 100 = 355.8 \text{ kPa}$$

and:

$$T_2 = \frac{T_2/T^*}{T_1/T^*} T_1 = \frac{0.8592}{0.3149} \times 283 = 772.2 \text{ K} = 499.2^\circ\text{C}$$

Therefore, the pressure and temperature of the air at the outlet to the duct are 355.8 kPa and 499.2°C respectively.

When the maximum amount of heat is transferred to the flow, the Mach number at exit becomes one, then the total temperature  $T_{02}$  will equal  $T_0^*$ . Hence in this case:

$$T_{02} = \frac{T_{01}}{T_{01}/T_0^*} = \frac{726.7}{0.6738} = 1078.5 \text{ K}$$

but  $q = C_p(T_{02} - T_{01}) = 1.007(1078.5 - 726.7) = 354.3 \text{ kJ/kg}$

it having been assumed that  $C_p = 1.007 \text{ kJ/kg}$ .

Also when the maximum amount of heat has been transferred:

$$p_2 = p^* = \frac{p_1}{p_1/p^*} = \frac{100.7}{0.2004} = 499.0 \text{ kPa}$$

and:  $T_2 = T^* = \frac{T_1}{T_1/T^*} = \frac{283}{0.3149} = 898.7 \text{ K} = 625.7^\circ\text{C}$

Therefore, when the maximum amount of heat is transferred, the heat added is 354.3 kJ for every kilogram of air flowing through the duct and under these circumstances the pressure and temperature of the air at the outlet to the duct are 499.0 kPa and 625.7°C respectively.

3-a) Fanno flow is the one-dimensional adiabatic flow in a constant area duct with friction effect. The mach number variation for supersonic inlet flow with the duct length is shown below. When  $L = L^*$  Mach number reaches unity at exit ; but when  $L$  increases there exist a normal shock wave in the duct which moves forward as  $L$  increases more and more; this is shown in Fig.4.

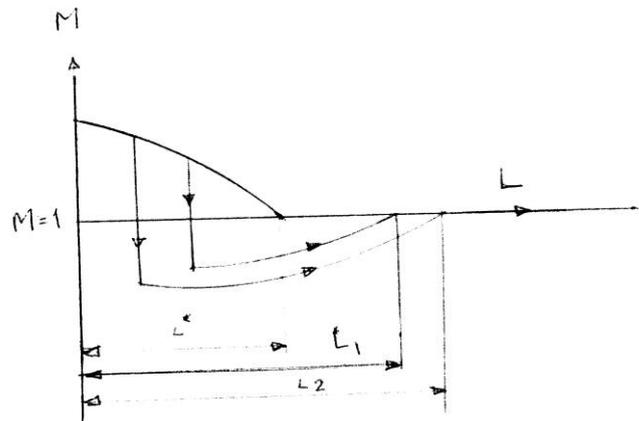


Figure 4

b) 1-The flow situation under consideration is shown in the figure 5 below

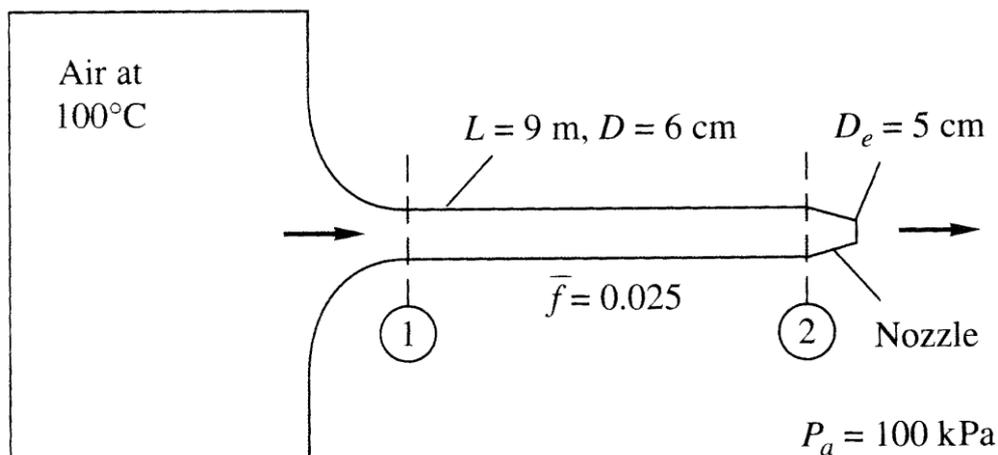


Figure 5

At the exit Mach number is 1 the area at exit is considered to the critical area so

$$T_e = T^* . \text{ Since } T_0 = 100 + 273 = 373 \text{ so } T_e = 310.8 \text{ K}$$

The exit speed  $V_e = 353.5 \text{ m/s}$

The mass flow rate  $\dot{m} = \rho_e v_e A_e \rightarrow \rho_e = 1.44 \text{ kg/s}$

$$P_e = \rho_e R T_e \quad \therefore P_e = 146.2 \text{ kPa} = P^* \rightarrow P_{0e} = 276.75 \text{ kPa}$$

The flow between section 2 and the exit is isentropic so  $P_{0e} = P_{02}$  and  $\frac{A_2}{A_e} = \frac{A_2}{A^*}$

$$\frac{A_2}{A^*} = 1.44 \rightarrow \text{from table } M_2 = 0.454 \text{ and } \frac{P_2}{P_{02}} = 0.868 \quad \therefore p_2 = 240.2 \text{ kPa}$$

$$\rightarrow \text{from table and for } M_2 = 0.454 \text{ we get } \frac{\bar{f}L_2^*}{D} = \frac{4fL_2^*}{D} = 1.57, \quad \frac{P_2}{P^*} = 2.36$$

$$\frac{4fL_1^*}{D} - \frac{4fL_2^*}{D} = \frac{4f(L_1^* - L_2^*)}{D} = \frac{4f(9)}{D} = 3.75 \rightarrow \frac{4fL_1^*}{D} = 5.32$$

$$\rightarrow \text{from table we get } M_1 = 0.3 \text{ and } \frac{P_1}{P^*} = 3.6191$$

$$P_1 = \left(\frac{P_1}{P^*} / \frac{P_2}{P^*}\right) \times p_2 \rightarrow P_1 = 368.35 \text{ kPa}$$

The flow between the tank and section 1 is isentropic, so the pressure inside the tank equal to the stagnation pressure at section 1  $P_{\text{tank}} = P_{01} = 392.07 \text{ kPa}$

4-a) The assumptions for derivation of the linearized velocity potential are :

i) The perturbations are very small  $\frac{u}{U_\infty} \ll 1, \frac{v}{U_\infty} \ll 1, \frac{w}{U_\infty} \ll 1.$

ii) Ranges of Mach number are  $0 < M_\infty < 0.8, M_\infty > 1.2$

iii) Flow is not hypersonic  $M_\infty < 5$

These assumptions fail at leading edge where assumption (i) is invalid, for transonic flow for which  $0.8 < M_\infty < 1.2$  ( assumption (ii) is invalid, and for hypersonic flow for which  $M_\infty > 5$ .

b)The lower critical Mach number ( $M_{\infty cr1}$ ) is the upstream ( non-disturbed ) flow Mach number at which the maximum local velocity on the airfoil reaches the speed of sound at one point only. This point is called the minimum pressure point or the point of maximum suction.

Evaluation the lower critical Mach number for airfoil with  $C_{p_{\text{imin}}} = -0.7$  .

The characteristic Mach number at point of maximum suction when the airfoil is traveling at  $M_{\infty cr1}$  is equal

1.0 i.e  $\lambda_c = 1.0$ , from Cristianowich diagram  $\lambda_i = 0.7577$

$$\text{From the equation } \lambda_i = \lambda_{\infty i} \sqrt{1 - C_{p_i}} \quad , \quad \lambda_{\infty i} = \frac{0.7577}{\sqrt{1 - (-0.7)}} = 0.581$$

From Cristianowich diagram the characteristic Mach number for the upstream ( non-disturbed ) flow is  $\lambda_{\infty c} = 0.638$  for which the  $M_{\infty cr1} = 0.8118$ .

5-a) It is known that  $\sin^{-1} 1/M_\infty < \beta < \pi/2$  . Consequently  $19.47^\circ < \beta < 90^\circ$  .

b) The situation is shown below in Fig.6

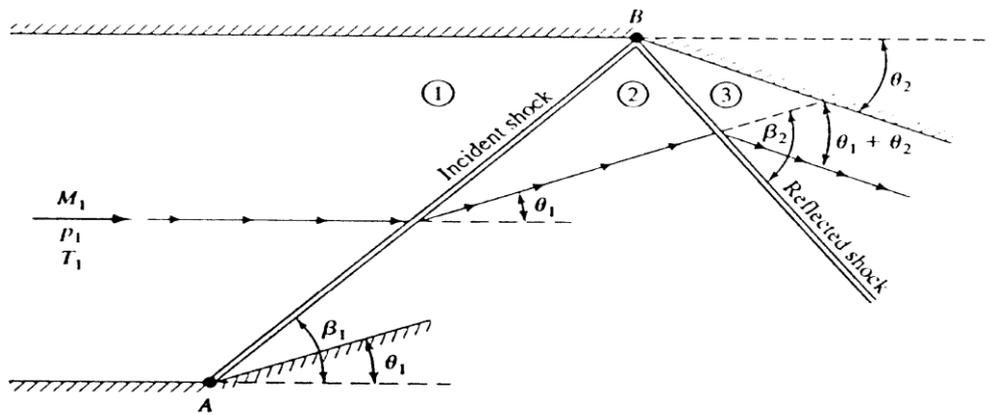


Figure 6

From the  $\theta$ - $\beta$ - $M$  diagram,  $\beta_1 = 31.2^\circ$ ,

$$M_{n1} = M_1 \sin \beta_1 = 3 \sin 31.2^\circ = 1.554$$

From Table A.2, for  $M_{n1} = 1.56$  (nearest entry),

$$\frac{p_2}{p_1} = 2.673, \quad \frac{T_2}{T_1} = 1.361, \quad M_{n2} = 0.6809$$

$$M_2 = \frac{M_{n2}}{\sin(\beta_1 - \theta_1)} = \frac{0.6809}{\sin(31.2 - 14)} = 2.30$$

The flow in region 2, at  $M_2 = 2.3$ , is deflected downward through the combined angle  $\theta_1 + \theta_2 = 14^\circ + 10^\circ = 24^\circ$ . From the  $\theta$ - $\beta$ - $M$  diagram for  $M = 2.3$  and  $\theta = 24^\circ$ ,  $\beta_2 = 52.5^\circ$ ,

$$M_{n2} = M_2 \sin \beta_2 = 2.3 \sin 52.5^\circ = 1.82$$

From Table A.2, for  $M = 1.82$ ,

$$\frac{p_3}{p_2} = 3.698, \quad \frac{T_3}{T_2} = 1.547, \quad M_{n3} = 0.6121$$

$$M_3 = \frac{M_{n3}}{\sin(\beta_2 - \theta_1 - \theta_2)} = \frac{0.6121}{\sin(52.5 - 24)} = \boxed{1.28}$$

$$p_3 = \frac{p_3}{p_2} \frac{p_2}{p_1} p_1 = (3.698)(2.673)(1) = \boxed{9.88 \text{ atm}}$$

$$T_3 = \frac{T_3}{T_2} \frac{T_2}{T_1} T_1 = (1.547)(1.361)(300) = \boxed{631.6 \text{ K}}$$

The process on the pressure deflection diagram is shown below in Figure 7 where the pressure change from  $p_1$  to  $p_2$  and finally to  $p_3$  at point 3.

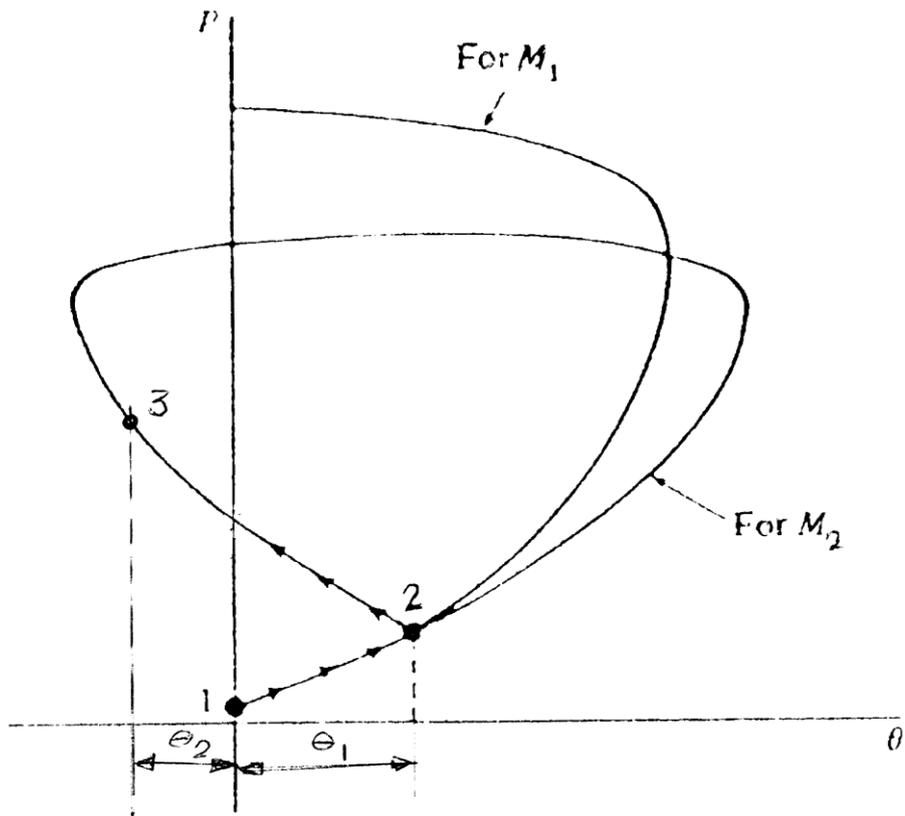


Figure 7 (reflection on pressure deflection diagram)

From  $\theta - \beta - M$  curves, for  $M_3 = 1.28$   $\theta_{\max} = 6.5$  degrees.