Benha University
Benha Faculty of Engineering
Electrical Engineering and Circuit Analysis(a) (E1101)
Dr.Wael Abdel-Rahman Mohamed

1st Term 2015-2016
Electrical Department
1st Year Electrical
Time: 3 Hrs

Model Answer

Question (1): **[12 Marks]**

a) Find the resistance seen by the ideal voltage source in the circuit in Fig.1.
b) If v_{ab} equals 400 V, how much power is dissipated in the 31 Ω resistor?

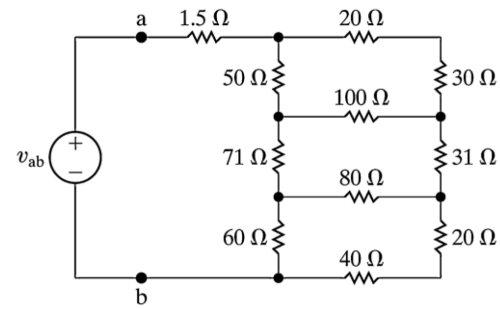


Fig.1

[a] Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$

$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$

$$R_3 = \frac{(100)(50)}{200} = 25\,\Omega$$

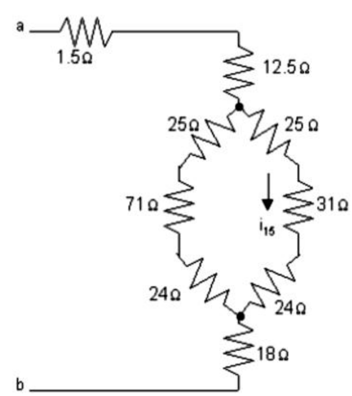
Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$

$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$

$$R_6 = \frac{(80)(60)}{200} = 24\,\Omega$$

Now redraw the circuit using the wye equivalents.



$$R_{ab} = 1.5 + 12.5 + \frac{(120)(80)}{200} + 18 = 14 + 48 + 18 = 80\,\Omega$$

[b] When $v_{ab} = 400$ V

$$i_g = \frac{400}{80} = 5 \text{ A}$$

$$i_{31} = \frac{48}{80}(5) = 3 \text{ A}$$

$$p_{31\Omega} = (31)(3)^2 = 279 \text{ W}$$

<u>Question (2):</u> [12 Marks]

For the circuit shown in Fig.2.

a) Write the node voltage equations needed to find the current i_Δ. (***Write both the main and the auxiliary equations***).

b) Write the mesh current equations needed to find the current i_Δ. (***Write both the main and the auxiliary equations***).

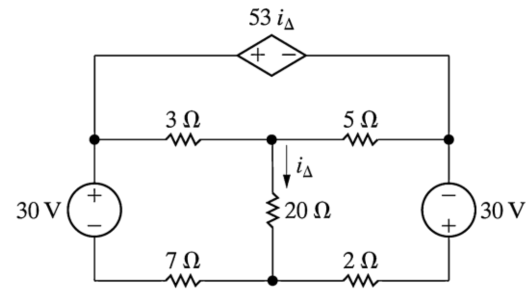
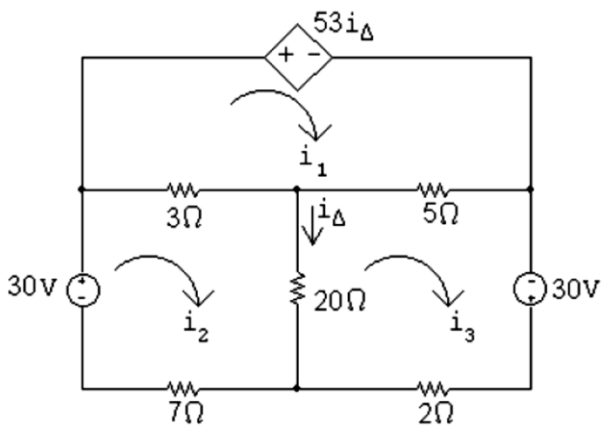


Fig.2

a) Solve by yourself.

b)



Mesh equations:

$$53i_\Delta + 8i_1 - 3i_2 - 5i_3 = 0$$

$$0i_\Delta - 3i_1 + 30i_2 - 20i_3 = 30$$

$$0i_\Delta - 5i_1 - 20i_2 + 27i_3 = 30$$

Constraint equations:

$$i_\Delta = i_2 - i_3$$

Question (3): **[12 Marks]**

a) Find the Norton equivalent circuit with respect to the terminals a,b for the circuit seen in Fig.3.

b) Find the maximum power that could be transferred to the load connected across the terminals a,b.

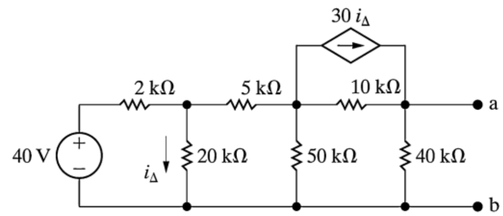
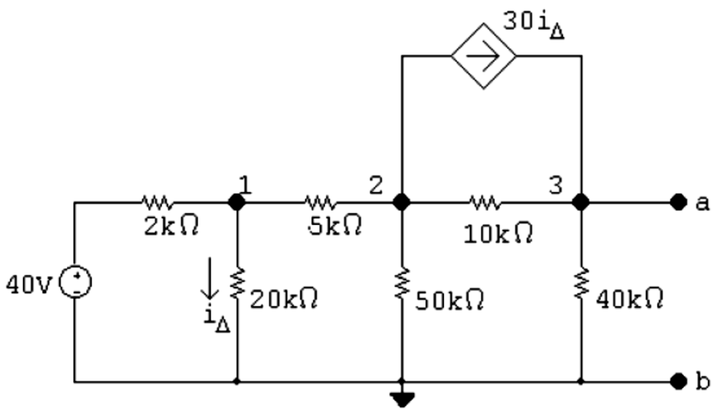


Fig.3

a) Norton equivalent circuit



The node voltage equations are:

$$\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30\frac{v_1}{20{,}000} = 0$$

$$\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30\frac{v_1}{20{,}000} = 0$$

In standard form:

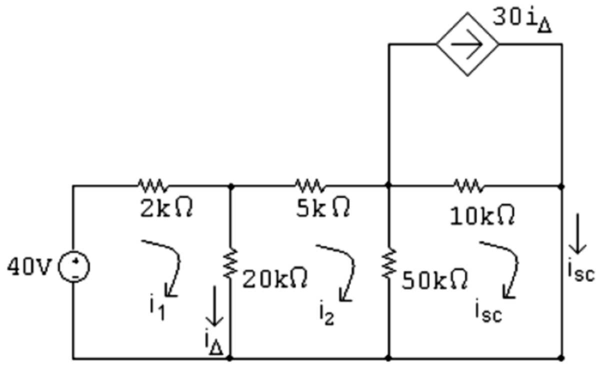
$$v_1\left(\frac{1}{2000} + \frac{1}{20{,}000} + \frac{1}{5000}\right) + v_2\left(-\frac{1}{5000}\right) + v_3(0) = \frac{40}{2000}$$

$$v_1\left(-\frac{1}{5000} + \frac{30}{20{,}000}\right) + v_2\left(\frac{1}{5000} + \frac{1}{50{,}000} + \frac{1}{10{,}000}\right) + v_3\left(-\frac{1}{10{,}000}\right) = 0$$

$$v_1\left(-\frac{30}{20{,}000}\right) + v_2\left(-\frac{1}{10{,}000}\right) + v_3\left(\frac{1}{10{,}000} + \frac{1}{40{,}000}\right) = 0$$

Solving, $v_1 = 24$ V; $v_2 = -10$ V; $v_3 = 280$ V

$V_{\text{Th}} = v_3 = 280$ V



The mesh current equations are:

$$-40 + 2000i_1 + 20{,}000(i_1 - i_2) = 0$$

$$5000i_2 + 50{,}000(i_2 - i_{sc}) + 20{,}000(i_2 - i_1) = 0$$

$$50{,}000(i_{sc} - i_2) + 10{,}000(i_{sc} - 30i_\Delta) = 0$$

The constraint equation is:

$$i_\Delta = i_1 - i_2$$

Put these equations in standard form:

$$i_1(22{,}000) + i_2(-20{,}000) + i_{sc}(0) + i_\Delta(0) = 40$$

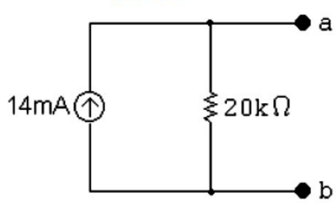
$$i_1(-20{,}000) + i_2(75{,}000) + i_{sc}(-50{,}000) + i_\Delta(0) = 0$$

$$i_1(0) + i_2(-50{,}000) + i_{sc}(60{,}000) + i_\Delta(-300{,}000) = 0$$

$$i_1(-1) + i_2(1) + i_{sc}(0) + i_\Delta(1) = 0$$

Solving, $i_1 = 13.6$ mA; $i_2 = 12.96$ mA; $i_{sc} = 14$ mA; $i_\Delta = 640\,\mu$A

$$R_{Th} = \frac{280}{0.014} = 20\ \text{k}\Omega$$



b) The maximum power transferred to the load = $(V_{th})^2/4R_{th}$ = 0.98 Watt

Question (4): **[12 Marks]**

The two op-amps in the circuit in Fig.4 are ideal, calculate v_{o1} and v_{o2}.

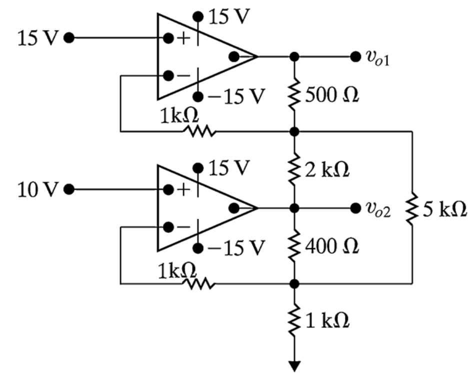
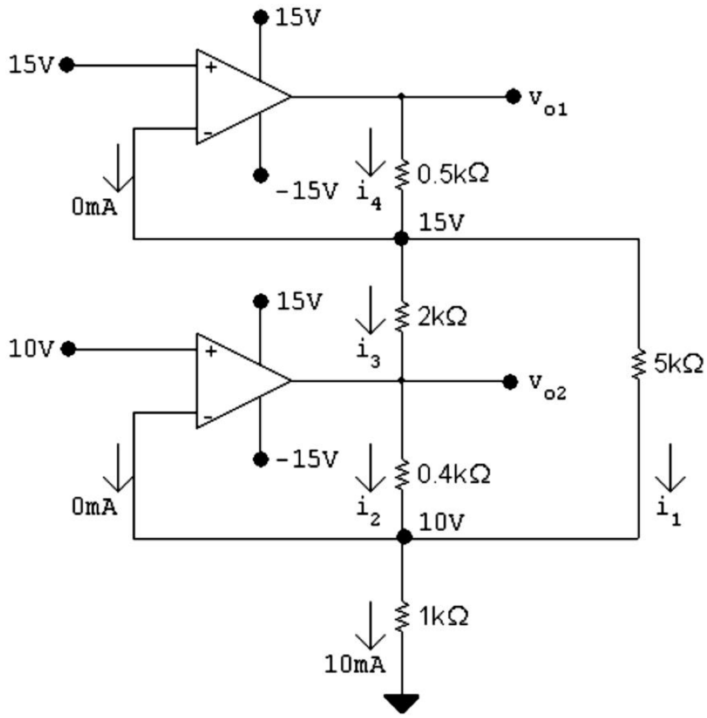


Fig.4

No current flows in the resistors in the feedback, so they are neglected.



$$i_1 = \frac{15-10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \text{ V}$$

$$i_3 = \frac{15-13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \text{ V}$$

Question (5): **[12 Marks]**

The switch in the circuit of Fig.5 has been in position **a** for a long time. At $t = 0$, it moves instantaneously to position **b**. For $t \geq 0^+$, find:

a) $v_o(t)$.
b) $i_o(t)$.
c) $v_1(t)$.
d) $v_2(t)$.
e) The energy trapped in the capacitors as $t \to \infty$.

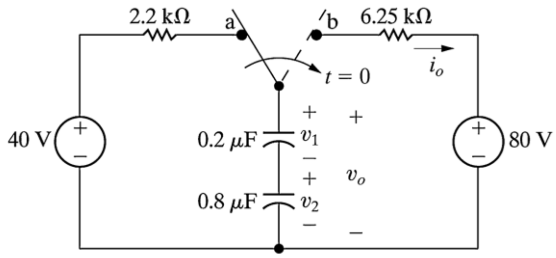
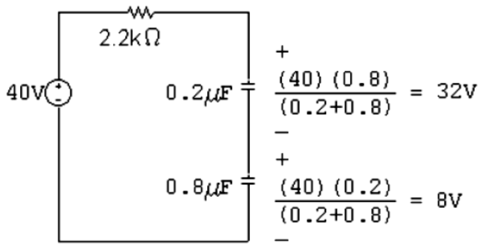
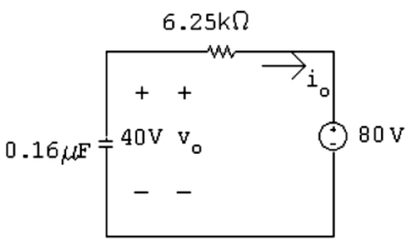


Fig.5

[a] $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16 \times 10^{-6})(6.25 \times 10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $$i_o = -C\frac{dv_o}{dt} = -0.16 \times 10^{-6}[40{,}000e^{-1000t}]$$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $$v_1 = \frac{-1}{0.2 \times 10^{-6}}\int_0^t -6.4 \times 10^{-3}e^{-1000x}\,dx + 32$$

$$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $$v_2 = \frac{-1}{0.8 \times 10^{-6}}\int_0^t -6.4 \times 10^{-3}e^{-1000x}\,dx + 8$$

$$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[e] $$w_{\text{trapped}} = \frac{1}{2}(0.2 \times 10^{-6})(64)^2 + \frac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512\,\mu\text{J}.$$

With best wishes