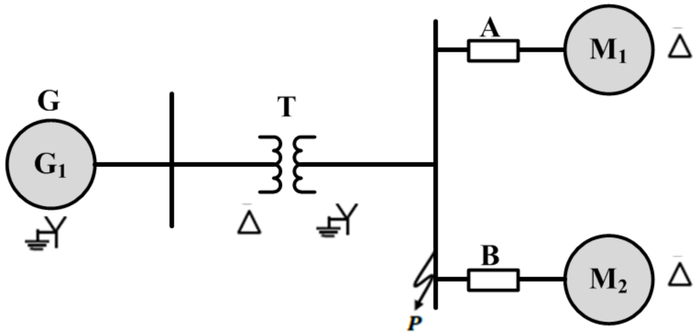
Benha University
Benha Faculty of Engineering
January, 2017
Solution (Regular)
Examiner: Mohamed Awaad ; Ph.D.

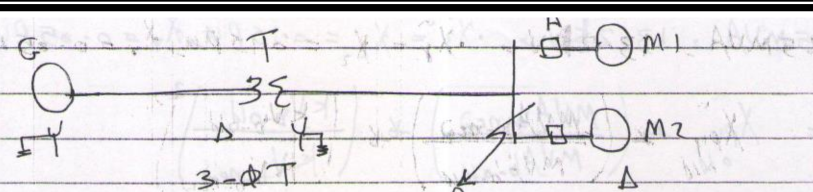
Electrical Department
B. Sc. Course Exam
Power System Analysis , E1437
Date: 15- 01-2017
Allowed Time: 3 Hours.

Question ❶ (*15 marks*)

(a) The Figure shows a 25 MVA, 13.2 kV generator with $X_d^{''} = X_2 = 0.15$ p.u., $X_0 = 0.05$ p.u. is connected through a transformer to a bus which supplies two identical motors with $X_d^{''} = X_2 = 0.2$ p.u., $X_0 = 0.05$ p.u. on base of 10 MVA , 6.9 kV . The three phase rating transformer is 30 MVA, 13.8 kV/6.9 kV with leakage reactance $X_l = 0.1$ p.u. The bus voltage at the motors is 6.9 kV. If the motors are operating at no load , calculate :

(a) The three phase short circuit current at point P in Amperes
(b) The rated short circuit and rated continuous current for two circuit breakers installed to protect motors





25 MVA, 13.2 kV, $X_d'' = X_2 = 0.15$ pu, $X_0 = 0.05$ pu

30 MVA, 13.8 kV/6.9 kV, $X_L = 0.1$ pu

identical motors: $X_d'' = X_2 = 0.2$ pu, $X_0 = 0.05$ pu, 10 MVA, 6.9 kV

The bus voltage at the motors is 6.9 kV. motors at no load.

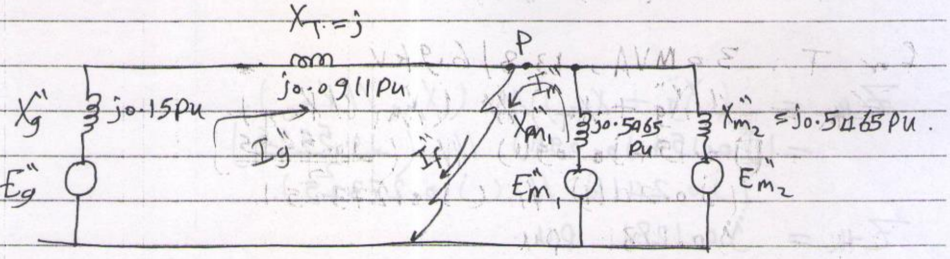
Calculate:-

(a) 3-Φ I_{sc} at P in A.

(b) .

∵ motors at no load ∴ $I_{nl} = 0$

"Sol"



Base MVA = 25

Base kV = 13.2 , $13.2 \times \frac{6.9}{13.8} = 6.6$ kV

$$\text{Base } I \left(\frac{MVA}{\sqrt{3}\, kV}\right) = \frac{25 \times 10^6}{\sqrt{3} \times 13.2 \times 10^3} , \frac{25 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3}$$

Base I = 1093.5 A , 2186.9 A ≅ 2187 A

(9)

G : 25 MVA , 13.2 kV , $X_g'' = X_2 = 0.15$ pu, $X_0 = 0.05$ pu.

$$X''_{pu_{new}} = X''_{pu_{old}} * \left(\frac{MVA_{b-new}}{MVA_{b-old}}\right) * \left(\frac{kV_{b-old}}{kV_{b-new}}\right)^2$$

$$X''_{pu_g} = 0.15\left(\frac{25}{25}\right)\left(\frac{13.2}{13.2}\right)^2 = 0.15 \text{ pu} = X_2$$

$$X_0 = 0.05\left(\frac{25}{25}\right)\left(\frac{13.2}{13.2}\right)^2 = 0.05 \text{ pu.}$$

For : T: 13.8/6.9 kV, 30 MVA, $X_L = 0.1$ p.u.

$$X_T = 0.1\left(\frac{25}{30}\right)\left(\frac{13.8}{13.2}\right)^2 = 0.0911 \text{ pu.}$$

for motors: 10 MVA, $X_d'' = X_2 = 0.2$ p.u, $X_0 = 0.05$ p.u, 6.9 kV.

$$X''_{d_{new}} = 0.2\left(\frac{25}{10}\right)\left(\frac{6.9}{6.6}\right)^2 = 0.5465 \text{ pu} = X_2, \quad X_{0_{pu}} = 0.1366 \text{ pu}$$

$$Z_{th} = (X_g'' + X_T) \,//\, (X_{m_1} // X_{m_2})$$
$$= (j0.15 + j0.0911) \,//\, \left(\frac{j0.5465}{2}\right)$$
$$(j0.2411) \,//\, (j0.27325).$$

$$Z_{th} = j0.1281 \text{ pu}$$

$V_{th} = V_f = 6.9$ kV at point P.

$$V_{th} = \frac{6.9}{6.6} = 1.045\angle 0 \text{ pu} \quad \text{as reference.}$$

$$I_f'' = \frac{V_{th}}{Z_{th}} = \frac{1.045\angle 0}{j0.1281} = -j\,8.158 \text{ pu.}$$

$\rightarrow$ 3-Φ SC at P

$$I_f'' = -j\,8.158 * 2187 = 17840.86 \text{ A.}$$

(10)

~~(b) the rated SC & rated Continuous currents for the two C.B installed to protect motors.~~

* rated SC for the two C.B.
  = Contribution of motors * 1.25 ; note $X''_M = X''_{m_1} // X''_{m_2} = \frac{X''_{m_1}}{2}$

$X''_M = j0.27325$ pu.

Contribution of 2 motors M1, M2 $= I''_M$

$$I''_m = I''_f * \left(\frac{X''_g + X_T}{X''_g + X_T + X''_m}\right) - I_L$$

∵ motors at no load ∴ $I_L = 0$

$$I''_m = -j8.158 * \frac{j(0.15+0.0911)}{j(0.15+0.0911+0.27325)} - 0$$

$$= -j\,8.158 * 0.4687 = -j3.824 \text{ pu.}$$

$$I''_m = -j3.824 * 2187 = 8363.17 \text{ A.}$$

$$I''_{m_1} = I''_{m_2} = I''_m * \frac{X''_{m_1}}{X''_{m_1} + X''_{m_2}} = \frac{1}{2} I''_m = -j1.912 \text{ pu}$$

$$I''_{m_1} = I''_{m_2} = -j4181.6 \text{ A.}$$

* rated ~~Continuous~~ SC currents for the two C.B installed to protect motors

$$I_{\text{rated } SC\ C.B.A} = I_{\text{rated Conti. } C.B.B} = \frac{I''_{m_1}}{0.8} = 1.25\, I''_{m_1}$$

$$= 1.25 * -j1.912 = -j2.39 \text{ pu}$$

$$= -j5226.93 \text{ A}$$

But rated continuous current for each C.B.

$$I^A_{C.B} = I^B_{C.B} = 1.25 * I_{\text{rated of motor}}$$

$$I^{A}_{\substack{C.B\\ rated}} = I^{B}_{\substack{C.B\\ rated}} = 1.25 * \frac{10*10^{6}}{\sqrt{3}*6.9*10^{3}} = \cancel{836.74}\ A$$

$$= 1.25 * 836.74$$

$$= 1045.92\ A$$

$$= 1.04592\ kA.$$

==============================

(b) Obtain the sequence network for the system shown in figure in case of a fault at point F. Then find the current in pu at the fault point for a single line to ground fault.

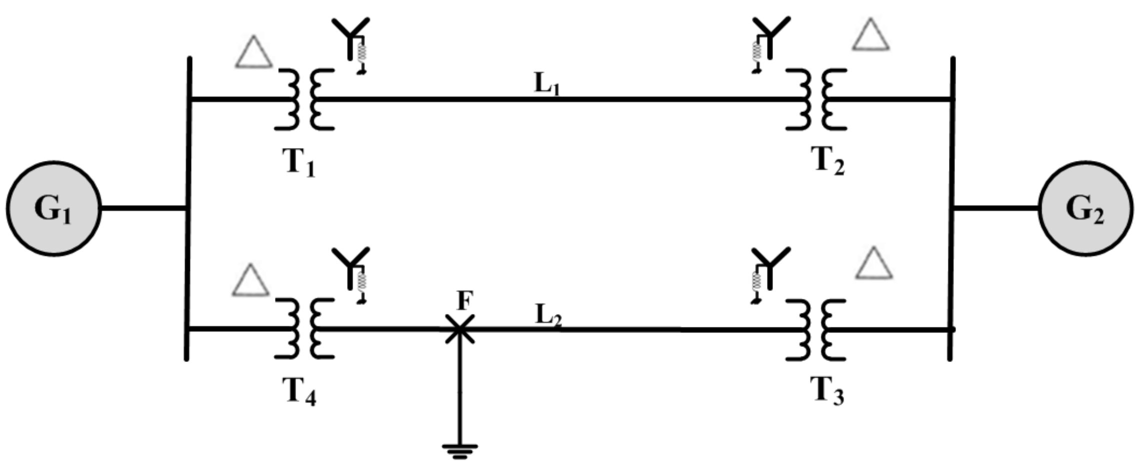
Assume the following data in pu on the same base are given:

$G_1 : x_1 = 0.2pu, x_2 = 0.12pu, x_0 = 0.06pu$

$G_2 : x_1 = 0.25pu, x_2 = 0.15pu, x_0 = 0.08pu$

$T_1, T_2, T_3, T_4 : x_1 = x_2 = x_0 = 0.2pu$

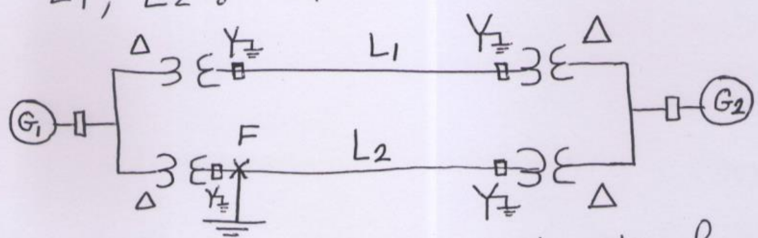
$L_1, L_2 : x_1 = x_2 = 0.15pu, x_0 = 0.3pu$



G_2 : $X_1 = 0.25$ pu, $X_2 = 0.15$ pu, $X_0 = 0.08$ pu

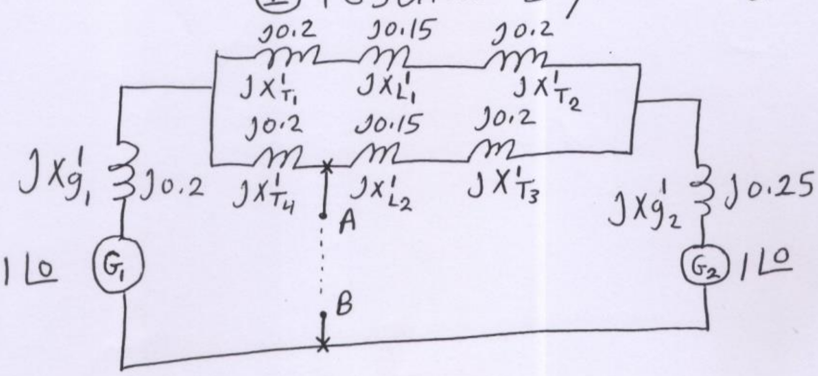
T_1, T_2, T_3, T_4 : $X_1 = X_2 = X_0 = 0.2$ pu

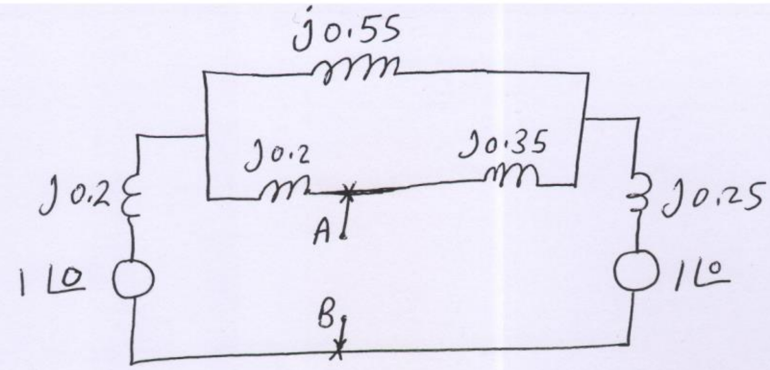
L_1, L_2 : $X_1 = X_2 = 0.15$ pu, $X_0 = 0.3$ pu

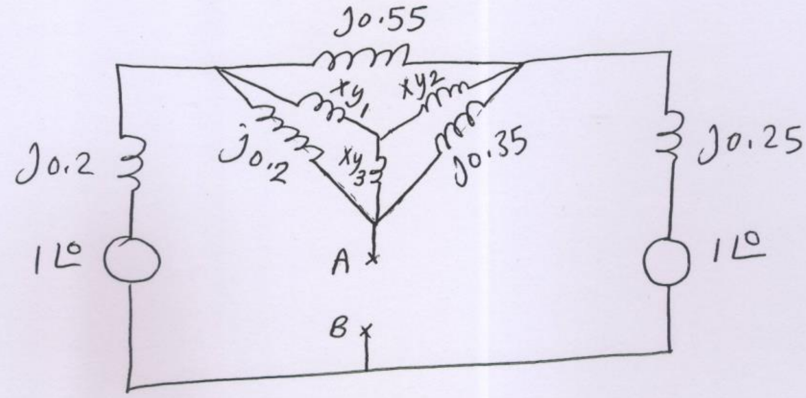


Obtain :- ① Sequence networks for the system.
② Current in P.u at the fault point for a single line-to-ground fault.

(I) Positive sequence network



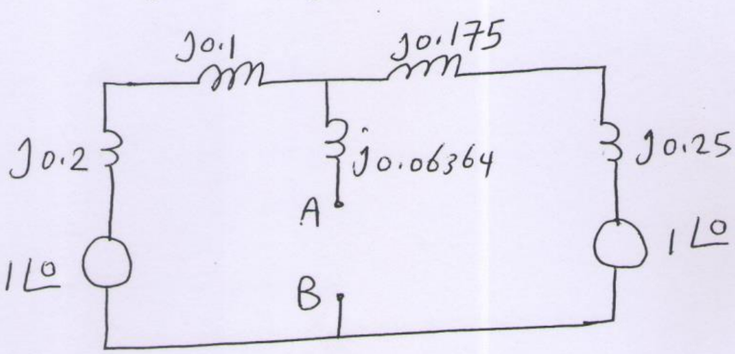


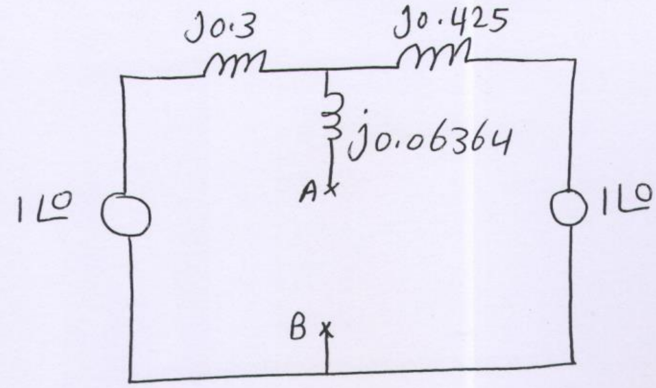


$$jXy_1 = \frac{j0.55 * j0.2}{j0.55 + j0.2 + j0.35} = j0.1$$

$$jXy_2 = \frac{j0.55 * j0.35}{j0.55 + j0.2 + j0.35} = j0.175$$

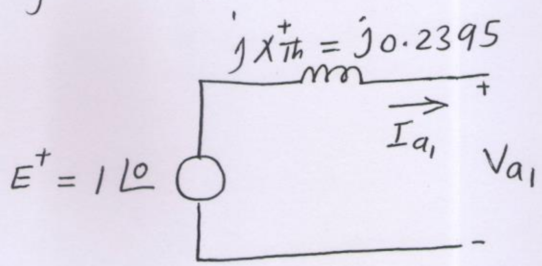
$$jXy_3 = \frac{j0.2 * j0.35}{j0.55 + j0.2 + j0.35} = j0.06364$$



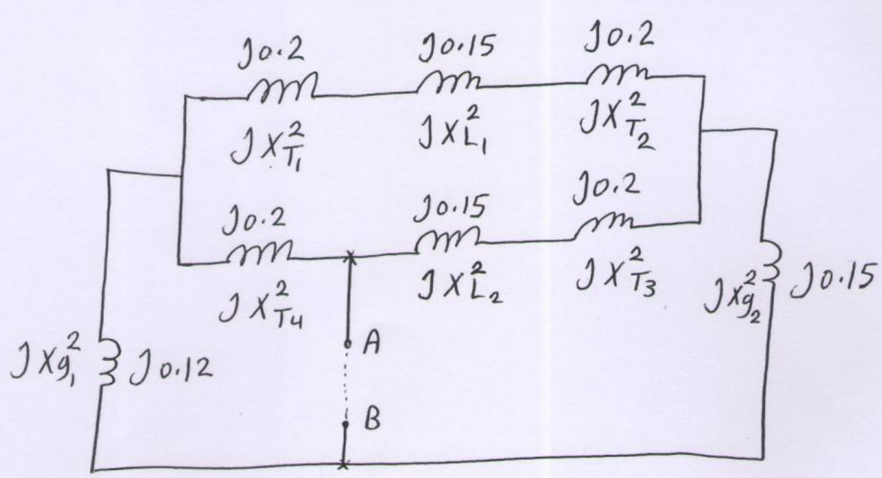


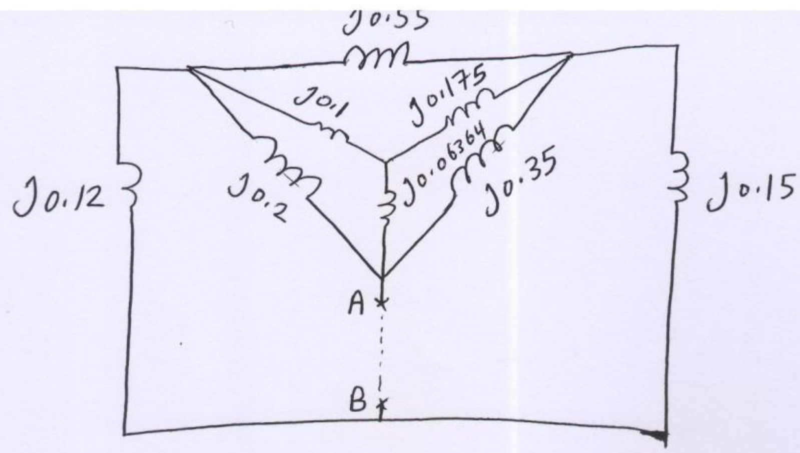
$$jX^{+}_{Th} = jX'_{Th} = j0.06364 + [j0.3 \,//\, j0.425]$$

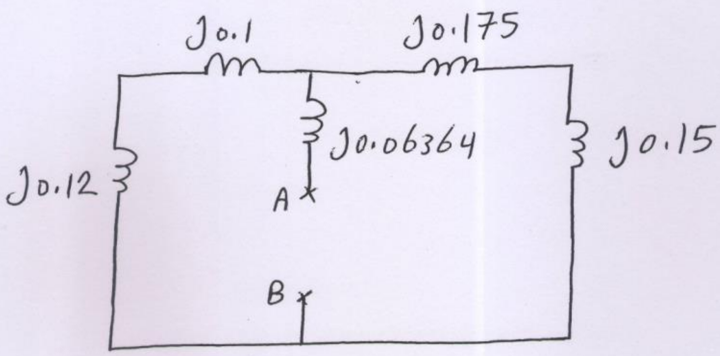
$$jX^{+}_{Th} = j0.2395 \text{ pu}$$

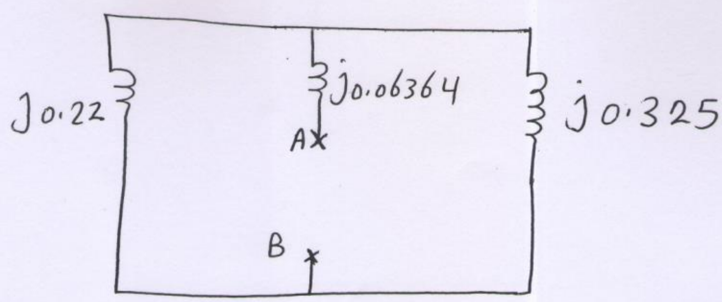


(II) Negative Sequence network



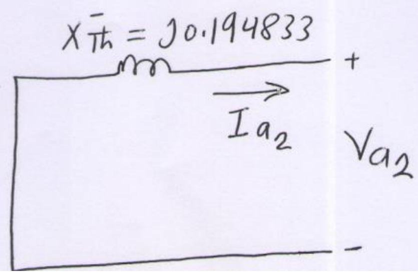




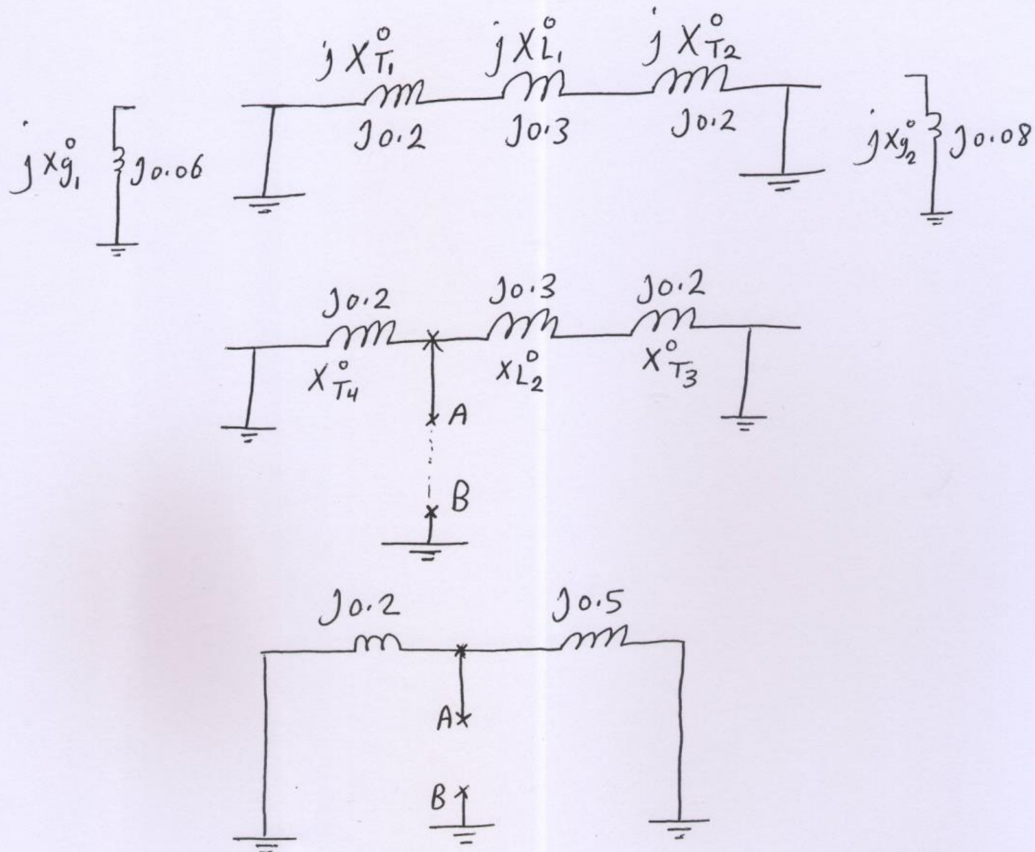


$$jX_{Th}^{-} = X_{Th}^{2} = j0.06364 + [\,j0.325 \,//\, j0.22\,]$$

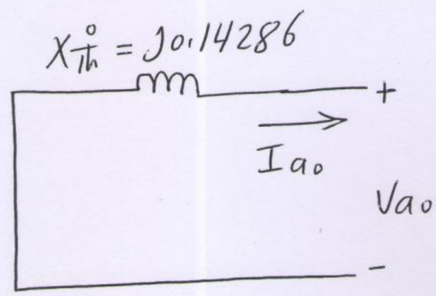
$$jX_{Th}^{-} = j0.194833 \text{ PU}$$



III - zero sequence network



$$jX^{o}_{Th} = (j0.2 \,//\, j0.5) = j0.14286 \text{ P.U}$$



$$V_a = V_{a1} + V_{a2} + V_{a0} = 0$$

$$I_{a0} = I_{a1} = I_{a2}$$

$$I_{a1} = \frac{E^+}{jX_{Th}^+ + jX_{Th}^- + jX_{Th}^0}$$

$$= \frac{1\angle 0}{j0.2395 + j0.194833 + j0.14286}$$

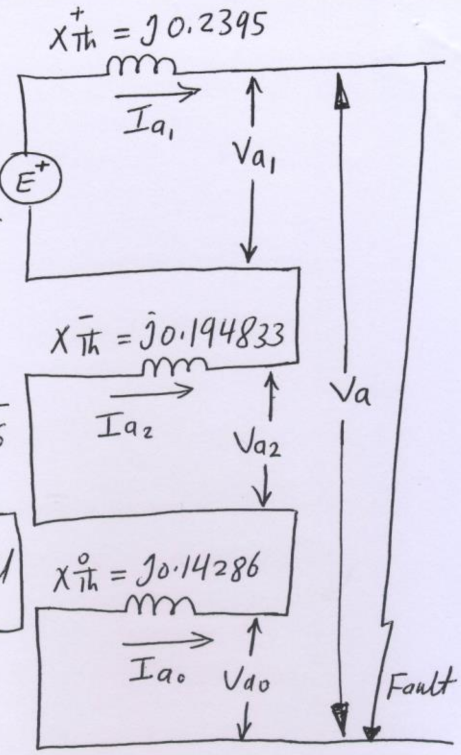
$$\boxed{I_{a1} = I_{a2} = I_{a0} = 1.7325\angle{-90}\ P.U}$$

$$I_a = 3I_{a1}$$

$$= 3 * 1.7325\angle{-90}$$

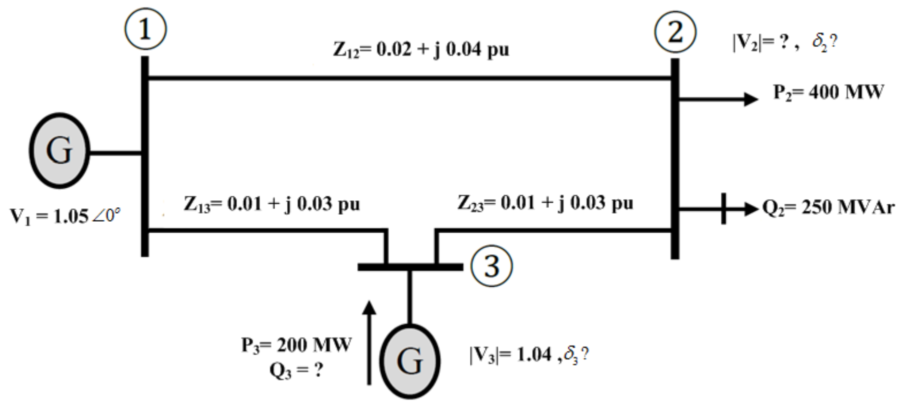
$$= 5.1976\angle{-90}\ P.u$$

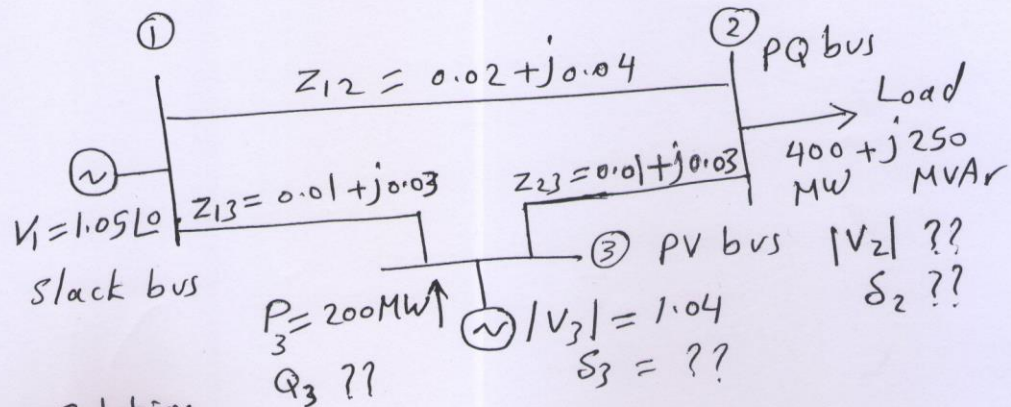
$$\boxed{I_a = 5.1976\angle{-90}\ P.u}$$



Question 2 (***30 marks***)

(a) The figure shows the single line diagram of a simple three power system with generation at buses 1 and 3. Transmission line impedances are marked in pu on 100 MVA base and line charging susceptances are neglected. Obtain the first iteration of V_2^1 , δ_2^1, Q_3^1 and δ_3^1 using the power flow solution by Gauss-Sidel method.





Solution

obtain the power flow solution i.e obtain

$V_2 = |V_2|\angle\delta_2$, $V_3 = |V_3|\angle\delta_3$

$S_3 = 200\text{ MW} + jQ_3$, $S_1 = P_1 + jQ_1$

$$y_{12} = \frac{0.02 - j0.04}{(0.02)^2 + (0.04)^2} = \frac{0.02(1 - j2)}{(0.02)^2(1+4)} = 10 - j20$$

$$y_{13} = \frac{0.01 - j0.03}{(0.01)^2(1+9)} = 10 - j30 = y_{23}$$

$Y_{11} = y_{12} + y_{13} = 20 - j50$, $Y_{22} = y_{12} + y_{23} = 20 - j50$

$Y_{33} = 20 - j60 = y_{13} + y_{23}$

$Y_{12} = -10 + j20$, $Y_{13} = -10 + j30$, $Y_{23} = -10 + j30$

$$V_2^{k+1} = \frac{1}{Y_{22}}\left[\frac{S_2^{*\,inj}}{V_2^{*k}} - \left(Y_{21}V_1 + Y_{23}V_3^{k}\right)\right]$$

$$S_2 = 400 + j250$$
$$= 4 + j2.5 \text{ p.u}$$
$$S_2^{inj} = -4 - j2.5 \qquad S_2^{inj*} = -4 + j2.5 \text{ p.u}$$
$$V_2^0 = 1.0\angle 0, \quad V_1 = 1.05\angle 0, \quad V_3^0 = 1\angle 0$$
$$V_2^1 = \frac{1}{20-j50}\left[\frac{-4+j2.5}{1} - (-10+j20)1.05 - (-10+j30)1.04\right]$$
$$= \frac{1+j2.5}{20(7.25)}\left[-4+j2.5+10.5-j21+10.4-j31.2\right]$$
$$= \frac{1+j2.5}{145}\left[16.9-j49.7\right] = \frac{1}{145}(141.15-j7.4)$$
$$= 0.97345 - j0.0514 \text{ p.u} = 0.9748\angle -3.022^\circ$$
$$Q_3^1 = -I_m\left[V_3^{0*}\left(Y_{31}V_1 + Y_{32}V_2^1 + Y_{33}V_3^0\right)\right]$$
$$= -I_m\left[1.04\angle 0\left((-10+j30)1.05 + (-10+j30).\right.\right.$$
$$\left.(0.97345-j0.0514) + (20-j60)1.04\right]$$
$$= -[10.92+j32.76 - 8.5202 + j30.9062$$
$$+ 21.632 - j64.896] = 1.23$$
$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{S_3^*}{V_3^{*k}} - \left(Y_{31}V_1 + Y_{32}V_2^1\right)\right]$$
$$S_3 = 2 + j1.23 \qquad \therefore S_2^* = 2 - j1.23$$

$$V_3^1 = \frac{1}{20-j60}\left[\frac{2-j1.23}{1.04} - \left((-10+j30)1.05\right.\right.$$
$$\left.+(-10+j30)(0.97345-j0.0514)\right]$$

$$= \frac{(1+j3)}{200}\left[1.9231 - j1.18269 + 10.5 - j31.5\right.$$
$$\left.+9.7345 - 1.542 - j0.514 - j29.2035\right]$$

$$= \frac{(1+j3)}{200}(20.6156 - j62.4002)$$

$$= \frac{207.8162 - j0.55339}{200} = 1.03908 - j.00277$$

Since V_3 is held constant at 1.04 p.u

∴ only the Imaginary part is retained

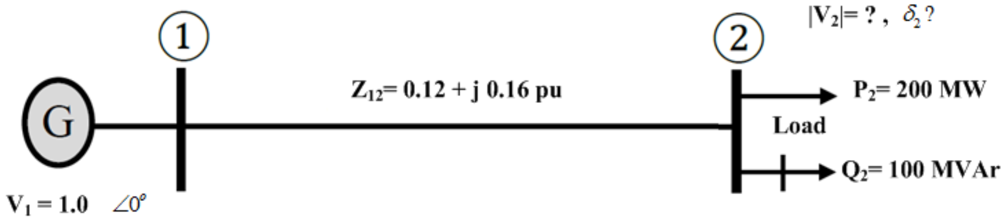
∴ Real part of $V_3 = \left[1.04^2 - 0.00277^2\right]^{1/2}$

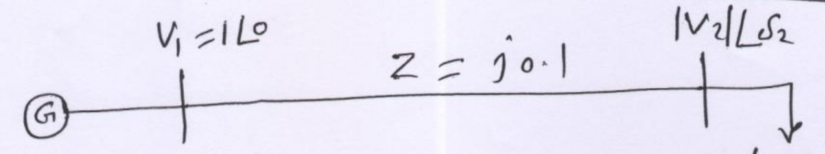
$= 1.039996 - j0.00277$

$= 1.04 \angle -0.1526^\circ$

===

(b) For the system shown in the figure a two bus power system , use Newten-Raphson Power flow method to determine the second iteration of voltage magnitude $|V_2|^2$ and its phase shift angle δ_2^2 at bus 2 and mismatch values of ΔP_2^2, ΔQ_2^2. Assume that bus 1 is the slack bus and $S_{base} = 100MVA$





load
$200MW + j\,100\,MVA$
$S_{base} = 100MVA$

$$Y_{BUS} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

$$B_{22} = -10, \quad B_{21} = 10, \quad G_{22} = G_{21} = 0$$

$$\Delta P_i = P_i^{inj} - \sum_{j=1}^{n} |V_i||V_j| \left[G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right]$$

$$\Delta Q_i = Q_i^{inj} - \sum_{j=1}^{n} |V_i||V_j| \left[G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right]$$

$$S_2^{inj} = -2 - j1 \quad p.u \qquad , \quad \delta_1 = 0, \; V_1 = 1\angle 0$$

$$\Delta P_2 = -2 - \left[V_2 V_1 B_{21} \sin(\delta_2 - \delta_1) + V_2^2 B_{22} \sin(\delta_2 - \delta_2) \right]$$

$$\Delta P_2 = -2 - 10 V_2 \sin\delta_2$$

$$\Delta Q_2 = -1 - \left[-V_2 V_1 B_{21} \cos(\delta_2 - \delta_1) - V_2 V_2 B_{22} \cos(\delta_2 - \delta_2) \right]$$

$$= -1 + 10 V_2 \cos\delta_2 - 10 V_2^2$$

$$J(x) = \begin{bmatrix} \dfrac{\partial \Delta P_2}{\partial \delta_2} & \dfrac{\partial \Delta P_2}{\partial V_2} \\ \dfrac{\partial \Delta Q_2}{} & \dfrac{\partial \Delta Q_2}{} \end{bmatrix}$$

$$\therefore J(x) = \begin{bmatrix} (-10V_2 \cos\delta_2) & (-10\sin\delta_2) \\ (-10V_2 \sin\delta_2) & (10\cos\delta_2 - 20V_2) \end{bmatrix}$$

$$[x^0] = \begin{bmatrix} \delta_2^0 \\ |V_2|^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$f(x^0) = \begin{bmatrix} \Delta P_2^0 \\ \Delta Q_2^0 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$J(x^0) = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}$$

$$x^1 = x^0 - [J(x^0)]^{-1}[f(x^0)]$$

$$\begin{bmatrix} \delta_2^1 \\ V_2^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} \quad \therefore \begin{bmatrix} \Delta P_2^1 \\ \Delta Q_2^1 \end{bmatrix} = \begin{bmatrix} -0.211 \\ -0.279 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^2 \\ V_2^2 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} +8.82 & -1.988 \\ -1.789 & +8.199 \end{bmatrix}^{-1} \begin{bmatrix} +0.211 \\ +0.279 \end{bmatrix}$$

$$J^{-1} = \frac{1}{68.7587} \begin{bmatrix} 8.199 & 1.789 \\ 1.988 & 8.82 \end{bmatrix} = \begin{bmatrix} 0.1192 & 0.026 \\ 0.0289 & 0.12827 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^2 \\ V_2^2 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 0.02515 + 0.00702 \\ 0.0061 + 0.0358 \end{bmatrix} = \begin{bmatrix} -0.2322 \\ 0.8581 \end{bmatrix}$$

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Benha Faculty of Engineering
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Examiner: Mohamed Awaad ; Ph.D.

Electrical Department
B. Sc. Course Exam
Power System Analysis , E1437
Date: 15- 01-2017
Allowed Time: 3 Hours.

$$\left[f(x)^2\right] = \begin{bmatrix} \Delta P_2^2 \\ \Delta Q_2^2 \end{bmatrix} = \begin{bmatrix} -0.02436 \\ -0.01288 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^3 \\ V_2^3 \end{bmatrix} = \begin{bmatrix} -0.2322 \\ 0.8581 \end{bmatrix} - \begin{bmatrix} +8.3501 & -2.3023 \\ -1.9756 & +7.4306 \end{bmatrix} \begin{bmatrix} +0.02436 \\ +0.01288 \end{bmatrix}$$

$$\left[J^2\right]^{-1} = \frac{1}{57.4978} \begin{bmatrix} 7.4306 & 1.9756 \\ 2.3023 & 8.3501 \end{bmatrix}$$

$$= \begin{bmatrix} 0.12927 & 0.03436 \\ 0.04004 & 0.14522 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^3 \\ V_2^3 \end{bmatrix} = \begin{bmatrix} -0.2322 \\ 0.8581 \end{bmatrix} - \begin{bmatrix} 0.12927 & 0.03436 \\ 0.04004 & 0.14522 \end{bmatrix} \begin{bmatrix} 0.02436 \\ .01288 \end{bmatrix}$$

$$= \begin{bmatrix} -0.2322 \\ 0.8581 \end{bmatrix} - \begin{bmatrix} 0.00359 \\ 0.002846 \end{bmatrix} = \begin{bmatrix} -0.23576 \\ 0.8552 \end{bmatrix}$$

$$\therefore \delta_2 = -13.515^\circ$$

$$\begin{bmatrix} \Delta P_2^3 \\ \Delta Q_2^3 \end{bmatrix} = \begin{bmatrix} 0.0014 \\ -0.0015 \end{bmatrix}$$

It may be close enough to be the solution of $\delta_2, |V_2|$

$$\therefore S_1 = V_1 \sum_{j=1}^{2} Y_{1j}^* V_j^* = V_1 Y_{11}^* V_1^* + V_1 Y_{12}^* V_2^*$$

$$= 1\left[j10 \times 1 + 1(-j10)(0.8552)\underline{/13.515^\circ}\right]$$

$$= j10\left[1 - 0.8315 - j0.19986\right]$$

$$= 1.9986 + j1.685 \text{ p.u}$$

$$S_1 = 199.86 \text{ MW} + j168.5$$

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Question 3 (***15 marks***)

(a) Draw the block diagram of LFC and AVR of an isolated power system.

(b) A two bus power system is shown in Figure (b). Incremental fuel costs of the two generators are given as: $\frac{dF_1}{dP_1} = 0.35P_1 + 41.0\$/\text{MWh}$, $\frac{dF_2}{dP_2} = 0.35P_2 + 41.0\$/\text{MWh}$, Loss expression is $P_L = 0.001(P_2 - 70)^2$ MW. Determine the optimal scheduling and power loss of the transmission link.

$P_L = 0.001(P_2 - 70)^2$ MW.

Determine the optimal scheduling and the Transmission link Power Loss of the

$$PL = 0.001(P_2 - 70)^2$$

$$\frac{\partial PL}{\partial P_2} = 0.002(P_2 - 70)$$

$$L_1 = 1 \quad , \quad L_2 = \frac{1}{1 - \frac{\partial PL}{\partial P_2}}$$

$$\therefore L_2 = \frac{1}{1 - 0.002 P_2 + 0.14}$$

$$L_2 = \frac{1}{1.14 - 0.002 P_2}$$

$$\lambda_1 = \lambda = L_1 \frac{df_1}{dP_1} = 0.35 P_1 + 41 \longrightarrow ①$$

$$\lambda_2 = \lambda = L_2 \frac{df_2}{dP_2} = \frac{0.35 P_2 + 41}{1.14 - 0.002 P_2} \longrightarrow ②$$

from ① and ②

$$\frac{0.35 P_2 + 41}{1.14 - 0.002 P_2} = 0.35 P_1 + 41$$

$$[1.14 - 0.002 P_2][0.35 P_1 + 41] = 0.35 P_2 + 41 \rightarrow ③$$

also we have the power balance equation

$$P_1 + P_2 = 300 + 70 + P_{Loss}$$

$$= 300 + 70 + 0.001(P_2 - 70)^2$$

$$= 370 + 0.001(P_2 - 70)^2$$

$$= 370 + 0.001[P_2^2 - 140 P_2 + 4900]$$

$$P_1 + P_2 = 370 + 0.001 P_2^2 - 0.14 P_2 + 4.9$$

$$P_1 = 370 + 0.001 P_2^2 - 0.14 P_2 + 4.9 - P_2$$

$$\therefore P_1 = 0.001 P_2^2 - 1.14 P_2 + 374.9 \rightarrow ④$$

from ④ in ③

$$[1.14 - 0.002 P_2] * [[0.35(0.001 P_2^2 - 1.14 P_2 + 374.9)] + 41]$$

$$= 0.35 P_2 + 41$$

$$(1.14 - 0.002P_2) * [3.5 * 10^{-4} P_2^2 - 0.399P_2 + 131.215 + 41]$$

$$= 0.35 P_2 + 41 \qquad (3)$$

$$(1.14 - 0.002P_2)(3.5*10^{-4} P_2^2 - 0.399 P_2 + 172.215) = 0.35 P_2 + 41$$

$$\therefore 3.99*10^{-4} P_2^2 - 0.45486 P_2 + 196.3251$$
$$- 7*10^{-7} P_2^3 + 7.98*10^{-4} P_2^2 - 0.34443 P_2$$
$$= 0.35 P_2 + 41$$

$$-7*10^{-7} P_2^3 + 1.197*10^{-3} P_2^2 - 1.1493 P_2 + 155.3251 = 0$$

$P_2 = 159.041355$, $\cancel{1181.1818}$ → refused, 1181.1818 → refused

$$\boxed{\therefore P_2 = 159.041355 \text{ MW}}$$

$$P_L = 0.001 [P_2 - 70]^2$$
$$= 0.001 [159.041355 - 70]^2$$

$$\boxed{P_L = 7.928363 \text{ MW}}$$

$\because P_1 + P_2 = 370 + P_{Loss}$

$P_1 = 370 + P_{loss} - P_2$

$= 370 + 7.928363 - 159.041355$

$$\boxed{P_1 = 218.887 \text{ MW}}$$

$\because \lambda_1 = \lambda_2 = \lambda = 0.35 P_1 + 41$

$= (0.35 * 218.887) + 41$

$= 117.61045 \ \$/\text{MWh}$

$$\boxed{\lambda = 117.61045}$$

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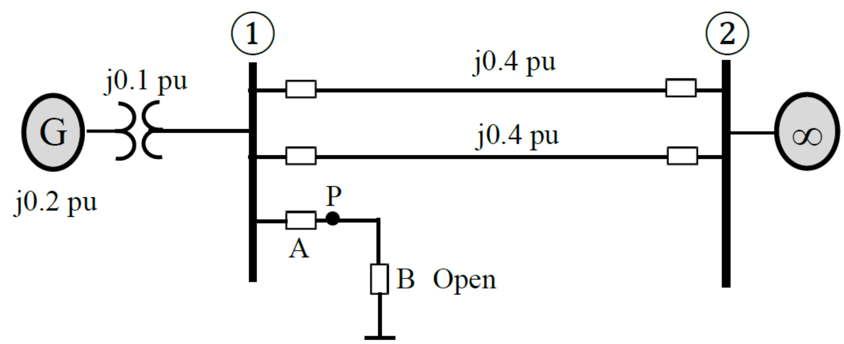
Question 4 (***30 marks***)

(a) A 50-Hz synchronous generator having inertia constant H = 5 MJ/MVA and a direct axis transient reactance $X_d^{'} = 0.2$ per unit is connected to an infinite bus through a purely reactive circuit as shown in the figure. Reactances are marked on the diagram on a common system base. The generator is delivering real power $P_e = 1$ per unit to the infinite bus at a voltage of V = 1 per unit.

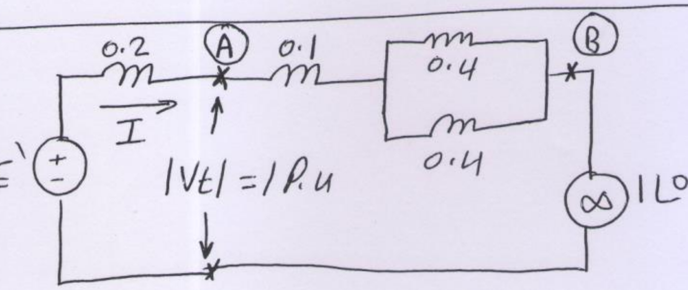


(a) Determine the power angle equation and the swing equation for the system shown in the figure.

(b) Calculate the critical clearing angle and the critical clearing time for the system shown in the figure when the system is subjected to a three-phase fault at point P on the short transmission line.

Q6:- 50 Hz, $H = 5\ MJ/MVA$, $X'_d = 0.2$ P.u
$P_e = 1$ P.u, both the generator terminal voltage and the infinite bus voltage are 1 P.u.

determine :- (a) Power angle equation and the swing equation.
(b) critical clearing angle and critical clearing Time.



(a) :- $X_{AB} = 0.1 + \frac{0.4}{2} = 0.3$ P.u

$$\frac{|V_t| . |V_\infty|}{X_{AB}} \sin\alpha = 1$$

$$\frac{1 * 1}{0.3} \sin\alpha = 1 \quad \therefore \alpha = \sin^{-1} 0.3$$

$$\alpha = 17.458^\circ$$

$$V_t = |V_t| \angle \alpha = 1 \angle 17.458$$

$$= 0.954 + j0.3 \text{ P.u}$$

$$E' = V_t + I\,(j0.2)$$

$$I = \frac{V_A - V_B}{j0.3}$$

$$= \frac{1\angle 17.458 - 1\angle 0}{j0.3}$$

$$= 1 + j\,0.1535$$

$$I = 1.012\angle 8.729$$

$$E' = (0.954 + j0.3) + (j0.2) * (1.012\angle 8.729)$$

$$E' = 1.05\angle 28.44 \quad P.u$$

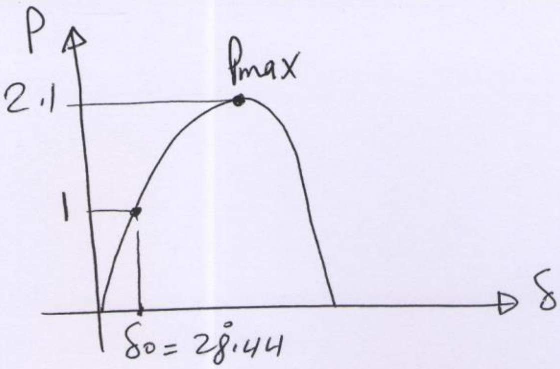
$$\delta_0 = 28.44^\circ$$

$$X = 0.2 + 0.1 + \frac{0.4}{2} = 0.5\ P.u$$

$$P_e = \frac{(1.05)(1)}{0.5}\sin\delta = 2.1\sin\delta \quad P.u$$

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = 1 - 2.1\sin\delta$$

Power angle equation and swing equation befor fault = after clearing fault but during fault



$$\delta_o = 28.44 = 0.496 \text{ rad}$$

(b) :-

$$\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_o)\sin\delta_o - \cos\delta_o\right]$$

$$= \cos^{-1}\left[(\pi - 2 * 0.496)\sin 28.44 - \cos 28.44\right]$$

$$\boxed{\delta_{cr} = 81.697^\circ = 1.426 \text{ rad}}$$

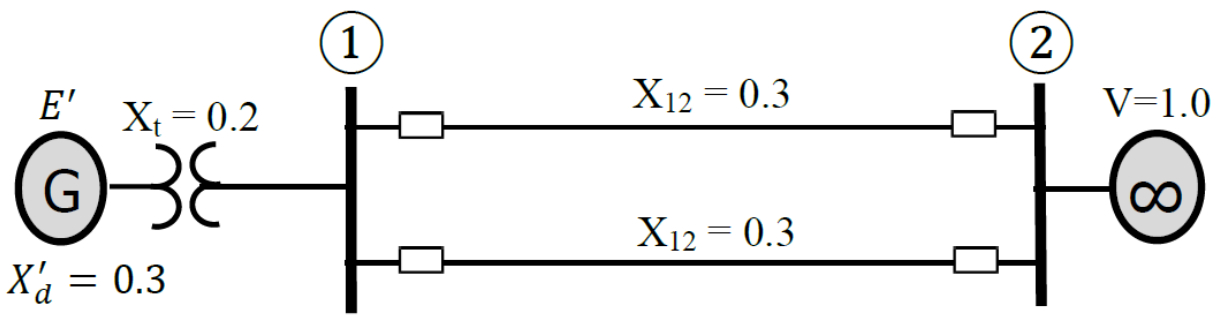
$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_o)}{\omega_s P_m}}$$

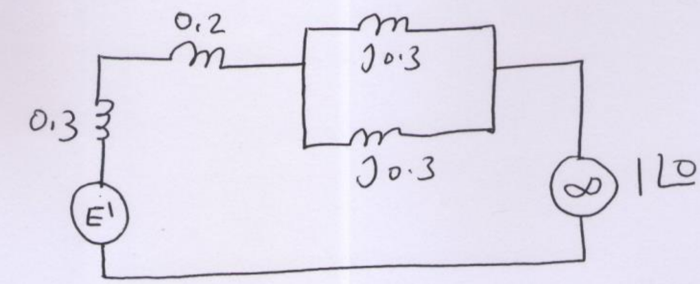
$$= \sqrt{\frac{4 * 5\,(1.426 - 0.496)}{(2\pi * 50) * 1}}$$

$$\boxed{t_{cr} = 0.2433 \text{ Sec}} \quad \text{no of cycles} = \frac{0.2433}{(1/50)} = 12$$

which is equivalent To acritical

(b) A 50-Hz synchronous generator having inertia constant H = 10 MJ/MVA and a transient reactance $X_d^{'}$ = 0.3 per unit is connected to an infinite bus through a purely reactive circuit as shown in the figure. The generator is delivering real power 0.6 per unit, 0.8 power factor lagging to the infinite bus at a voltage of V=1 per unit. Assume per unit damping power coefficient is D=0.138. Consider a small disturbance of Δδ =8° degrees. For example the breakers open and then quickly close. Obtain equations describing the motion of the rotor angle and the generator frequency.





$$X = 0.3 + 0.2 + \frac{0.3}{2} = 0.65 \text{ P.U}$$

$$E' = V_{\infty} + jIX.$$

$$S = \frac{P}{\cos\phi} = \frac{0.6}{0.8} \angle \cos^{-1} 0.8 = 0.75 \angle 36.87^{\circ}$$

$$I = \frac{S^*}{V^*} = \frac{0.75 \angle -36.87^{\circ}}{1 \angle 0} = 0.75 \angle -36.87^{\circ}$$

$$E' = V + JX \cdot I$$

$$= 1\angle 0 + J(0.65) * 0.75 \angle -36.87$$

$$E' = 1.35 \angle 16.79^\circ$$

$$\boxed{\delta_o = 16.79^\circ}$$

P_s = Synchronizing Power Coefficient

$$= P_{max} \cos \delta_o$$

$$= \frac{(1.35)(1)}{0.65} \cos 16.79$$

$$\boxed{P_s = 1.9884}$$

$$W_n = \sqrt{\frac{\pi f_o P_s}{H}} = \sqrt{\frac{\pi * 50 * 1.9884}{10}}$$

$$\boxed{W_n = 5.5887 \text{ rad/sec}}$$

$$\zeta = \frac{D}{2} \sqrt{\frac{\pi f_o}{H P_s}} = \frac{0.138}{2} \sqrt{\frac{\pi * 50}{10 * 1.9884}}$$

$$\boxed{\zeta = 0.19394}$$

$$\frac{d^2 \Delta\delta}{dt^2} + 2\zeta w_n \frac{d\Delta\delta}{dt} + w_n^2 \Delta\delta = 0$$

$$s^2 + 2\zeta w_n s + w_n^2 = 0$$

$$\frac{d^2 \Delta\delta}{dt^2} + 2.1677 \frac{d\Delta\delta}{dt} + 31.233 = 0$$

$$w_d = w_n \sqrt{1 - \zeta^2}$$

$$= 5.5887 \sqrt{1 - (0.19394)^2}$$

$$\boxed{w_d = 5.4826 \text{ rad/sec}}$$

$$f_d = \frac{w_d}{2\pi} = \frac{5.4826}{2\pi} = 0.8726 \text{ Hz}$$

$$\because \delta = \delta_0 + \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta w_n t} \sin(w_d t + \theta)$$

$$\because \theta = \cos^{-1}(\zeta) = \cos^{-1}(0.19394)$$

$$\theta = 78.817^\circ$$

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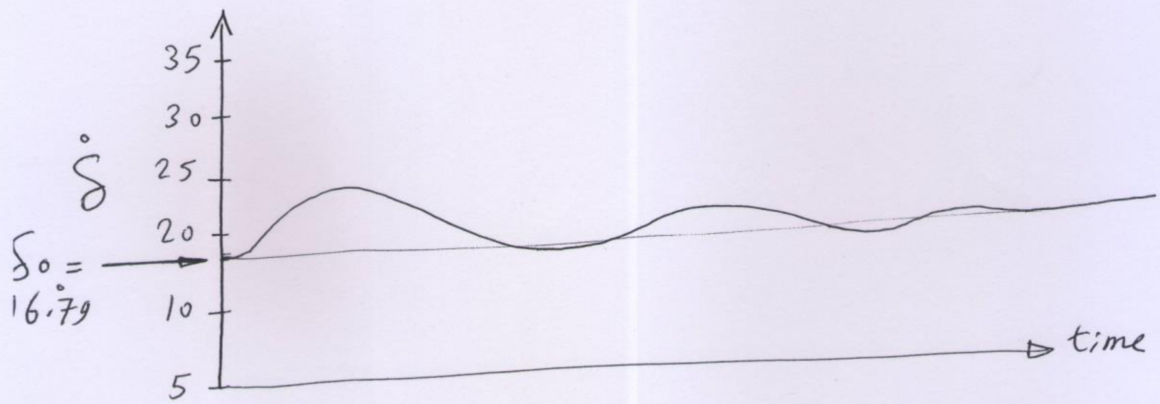
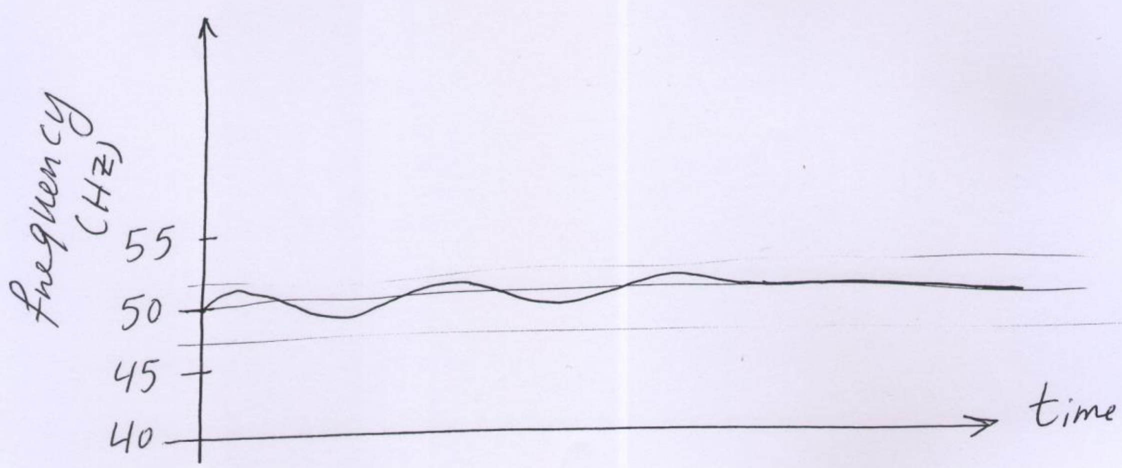
$$\Rightarrow \delta = 16.79 + 8.1548\, e^{-1.084} \sin(5.4826t + 78.817^\circ)$$

$$\because f = f_o - \frac{\omega_n \Delta\delta_o}{2\pi\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t} \sin \omega_d t$$

$$\Rightarrow f = 50 - 0.1266\, e^{-1.084} \sin 5.4826t$$





===========================Good Luck ===========================