Banha University | Second Term 2012/2013
Banha Faculty of Engineering | First Year Regular
Department of Basic Engineering Sciences

Solutions to the Questions For Written Term-Examination

Subject: Physics B1032 | Allowed Time: 3 Hours

Answer all questions | No. of Questiona:5 | No. of pages:2

Solution of Question 1

a) A 2 m long wire having a mass of 0.1 kg is fixed at both ends. The tension in the wire is 20 N. A node is observed at a point 0.4 m from one end. (i) What frequencies of the first three allowed modes of vibration? (ii) It is required to decrease the second allowed frequency by 3Hz, What is the tension in this case?

Answer

(i) Mass per unit length $\mu = m/L = 0.1/2 = 0.05$ kg/m

The velocity of the wave in the string:

$$V = \sqrt{F/\mu} = \sqrt{20/0.05} = 20 \text{ m/s}$$

The frequencies of allowed modes of vibration:

$$f_m = mv/2L$$
$$= m(20)/(2x2) = 5m \text{ Hz}$$

First mode m = 1, $f_1 = 5$ Hz
Second mode m = 2, $f_2 = 2 f_1 = 10$ Hz
Third mode m = 3, $f_3 = 3 f_1 = 15$ Hz

At resonance, the length of the string:

$$L = m\lambda/2$$

The position of nodes:

$$X_{node} = n\lambda/2$$

Note that n < m

Dividing the two equations:

$$L/x_{node} = m/n$$

$$m/n = 2/0.4 = 5$$

For n = 1, then m = 5. In this case the string is vibrating in fifth mode. $f_5 = 5 f_1 = 25$ Hz

(ii) $f_2 = 10 - 3 = 7$ Hz

$$f_2 = 2 v/2L = v/L$$

$v = f_2 L = 7x2 = 14$ m/s

$v = \sqrt{F/\mu}$

The tension force:

$F = v^2\mu = (14)^2(0.05) = 9.8$ N

b) Derive an expression for the intensity of sound wave in terms of density of medium ρ, sound velocity v, angular frequency ω and displacement amplitude S_m.

Answer

Consider an element of the medium of thickness Δx,

mass $\Delta m = \rho A \Delta x$

oscillating with displacement $S(x, t) = S_m \cos(kx - \omega t)$

The longitudinal velocity $v_x (x, t) = \partial S/\partial t$

$= - \omega S_m \sin(kx - \omega t)$

The kinetic energy of oscillation:

$KE = ½\, m\, v_x^2$

$= ½\, (\rho A \Delta x)\omega^2 S_m^2 \sin^2 (kx - \omega t)$

Since The average of $\sin^2 (kx - \omega t) = ¼$

The average kinetic energy = $¼(\rho A \Delta x)\omega^2 S_m^2$

The average kinetic energy = The average potential energy

The average total energy = 2 x The average kinetic energy

$= ½\, (\rho A \Delta x)\omega^2 S_m^2$

The average power P = energy/ Δt

$= ½\, \rho A(\Delta x/\Delta t)\omega^2 S_m^2$

Since $\Delta x/\Delta t = v$ the sound wave velocity, then

$$P = ½ \rho A v \omega^2 S_m^2$$

The sound intensity $I = P/A$

$$= ½ \rho v \omega^2 S_m^2$$

Solution to Question 2

a) Two pipes are of the same length. The first pipe has open ends, while the second pipe has one end close and the other end open. If a beat frequency of 10 Hz is heard, what is the length of the pipes. [velocity of sound = 343 m/s]

Answer

The resonance frequencies for the open pipe:

$$f_{open} = m_{open} v/2L$$

The resonance frequencies for the closed pipe:

$$F_{closed} = m_{closed} v/4L$$

The beat frequency $f_b = f_{open} - f_{closed}$

$$= m_{open} v/2L - m_{closed} v/4L$$

$$= (m_{open} - m_{closed}/2)\, v/2L$$

The length of the tube is

$$L = (m_{open} - m_{closed}/2)\, v/2f_b$$

$$= (m_{open} - m_{closed}/2)\, (343/2x10)$$

$$= (m_{open} - m_{closed}/2)\, x17.15$$

$m_{open} > m_{closed}/2$

m_{closed} is an odd number.

For $m_{closed} = 1$, $m_{open} = 1$, and, then $L = 8.575$ m

For $m_{closed} = 3$, $m_{open} = 2$, and, then $L = 8.575$ m

For $m_{closed} = 5$, $m_{open} = 3$, and, then $L = 8.575$ m

b) Derive an expression for the intensity of interference pattern on the screen of Young's double slit interference experiment.

Answer

In the Young's double slit interference experiment, assume that

slit separation distance = d

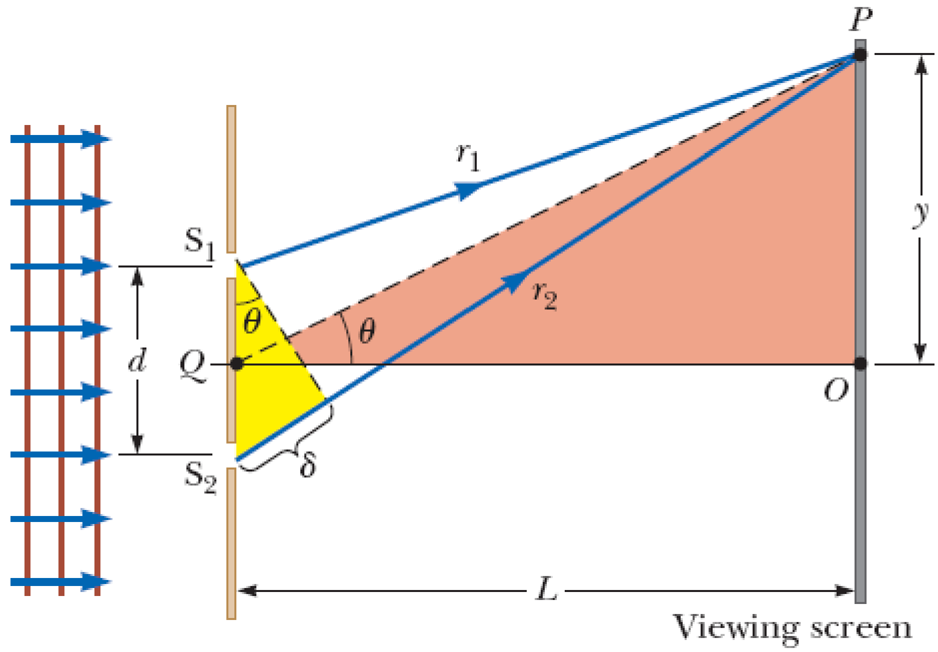
distance from slits to screen = L

wavelength of monochromatic light = λ

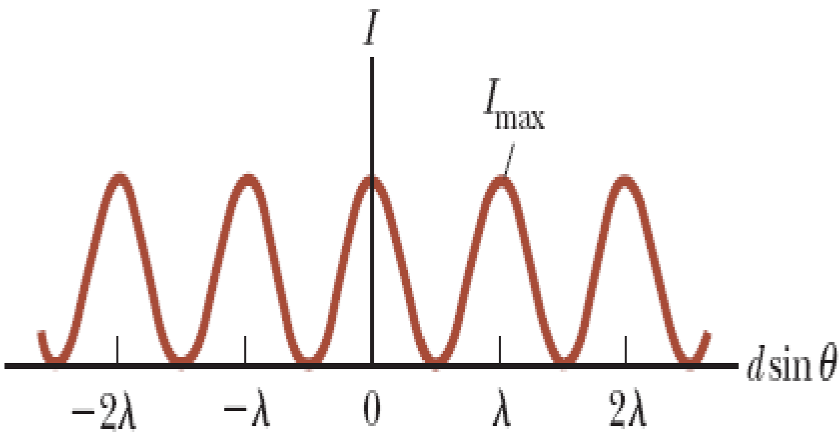
Any point on the screen receives light waves from each slit.

The path difference between the two waves is

$\delta = |r_2 - r_1| = d \sin\theta$



The phase difference $\Phi = (2\pi/\lambda)\delta = (2\pi/\lambda)\, d \sin\theta$
The electric field wave from slit 1 reaching a point P (at x = 0) on screen
$E_1 = E_o \sin \omega t$
The electric field wave from slit 2 reaching the same point P (at x = 0) on screen
$E_2 = E_o \sin (\omega t + \Phi)$
Applying the principle of superposition, the resulting wave is
$E = E_1 + E_2$
$= E_o [\sin (\omega t + \Phi) + \sin \omega t]$
$= 2 E_o \cos \Phi/2 \sin (\omega t + \Phi/2)$
The intensity of light is directly proportional to the square of the electric field
$I \propto E^2 = 4 E_o^2 \cos^2 \Phi/2 \sin^2 (\omega t + \Phi/2)$
The average intensity is then
$I \propto 2 E_o^2 \cos^2 \Phi/2$ (1)
Where the average of $\sin^2 (\omega t + \Phi/2)$ is ½.
The intensity at the center of the experiment where $\Phi = 0$ is
$I_o \propto 2 E_o^2$ (2)
Dividing equation (2) by equation (1), then the intensity
$I = 2 E_o^2 \cos^2 \Phi/2$



Bright fringes when the intensity is maximum.
$\cos \Phi/2 = 1$
$\Phi = 2m\pi$
$(2\pi/\lambda) d\sin\theta = 2m\pi$
$d\sin\theta = m\lambda$
$dy/L = m\lambda$
$y_m = m\lambda L/d$

Dark fringes when the intensity is minimum.
$\cos \Phi/2 = 0$
$\Phi = (2m + 1)\pi$
$(2\pi/\lambda) d\sin\theta = (2m + 1)\pi$
$d\sin\theta = (m + ½)\lambda$
$dy/L = (m + ½)\lambda$
$y_m = (m + 1/2)\lambda L/d$

<u>Solution to Question 3</u>

a) A thin layer of cryolite (n = 1.32) is applied to a camera lens of index of refraction of 1.5. The coating is designed to minimize reflections of blue light of wavelength 450. What minimum thickness is required?

<u>Answer</u>

The state is air – film – glass.
Index of refraction of film n = 1.32
Wavelength of light λ = 450 nm
If the film thickness is t,
the condition for minimum reflections is
$2nt = (m + ½)\lambda$

For minimum thickness m = 0,
Then, $t_{min} = \lambda/4n = 450x10^{-9}/4*1.32 = 8.523x10^{-8}$ m = 85.23 nm

b) A diffraction pattern is formed on a screen 150 cm away from a 0.3 mm wide slit. Monochromatic light of wavelength 546 nm is used. Calculate the fractional intensity I/I_o at a point on the screen 4 mm from the center of the principal maximum. What is the position of the next point on the screen having the same fractional; intensity?

<u>Answer</u>

L = 150 cm
a = 0.3 mm
λ = 546 nm
y = 4 mm

Phase difference $\beta = (2\pi/\lambda)\ a \sin\theta = (2\pi/\lambda)\ ay/L$
$= (2\pi)*(0.3x10^{-3})*(4x10^{-3})/(546x10^{-9})*(1.5)$
$= 9.206$ rad

The fractional intensity $I/I_o = \sin^2\beta/\beta^2 = \sin^2(9.206)/(9.206)^2$
$= 5.56x10^{-4}$

Solutions to Question 4

a) One mole of an ideal gas does 3000 J of work on its surroundings as it expands isothermally to a final pressure of 1 atm. And volume of 25 L. Determine (i) the initial volume and (ii) the temperature of the gas. [R = 8.31 J/mol.K] [1 atm = $1.0135\ x10^5$ Pa]

<u>Answer</u>

(ii) For ideal gas:

$p_fV_f = nR\ T$
$T = p_fV_f/nR$
$= (1.0135x10^5)*(25x10^{-3})/(1)(8.31)$
$= 304.9$ K

(i) For isothermal process, the work done:

$W = nRT\ \ln(V_f/V_i)$
$V_f/V_i = e^{W/nRT}$
$= e^{3000/(8.31)(304.9)}$
$= 3.27$

$V_i = V_f/3.27 = 25/3.27 = 7.65$ L

b) A 2 mole of diatomic ideal gas expands adiabatically from pressure of 5 atm and a volume of 12 L to a final volume of 30 L. (i) What are the final pressure, the initial temperature and the final temperature? (ii) Find Q, W and ΔU.

Answer

(i) For adiabatic process:

$$p_i V_i^{\gamma} = p_f V_f^{\gamma}$$

$$p_f = p_i (V_i/V_f)^{\gamma}$$

$$= (5)(12/30)^{1.4}$$

$$= 1.386 \text{ atm}$$

For ideal gas:

$$p_i V_i = nRT_i$$

$$T_i = p_i V_i / nR$$

$$= (5\text{x}1.0135\text{x}10^5)(12\text{x}10^{-3})/(2)(8.31)$$

$$= 365.88 \text{ K}$$

For ideal gas:

$$P_f V_f = nRT_f$$

$$T_f = p_f V_f / nR$$

$$= (1.386\text{x}1.0135\text{x}10^5)(30\text{x}10^{-3})/(2)(8.31)$$

$$= 253.56 \text{ K}$$

(ii) For adiabatic process: $Q = 0$

$$\Delta U = nC_V(T_f - T_i)$$

$$= (2)(5/2)(8.31)(253.56 - 365.88)$$

$$= -4666.9 \text{ J}$$

From the first law of thermodynamics:

$$W = Q - \Delta U$$
$$= 0 - (-4666.9)$$
$$= 4666.9 \text{ J}$$

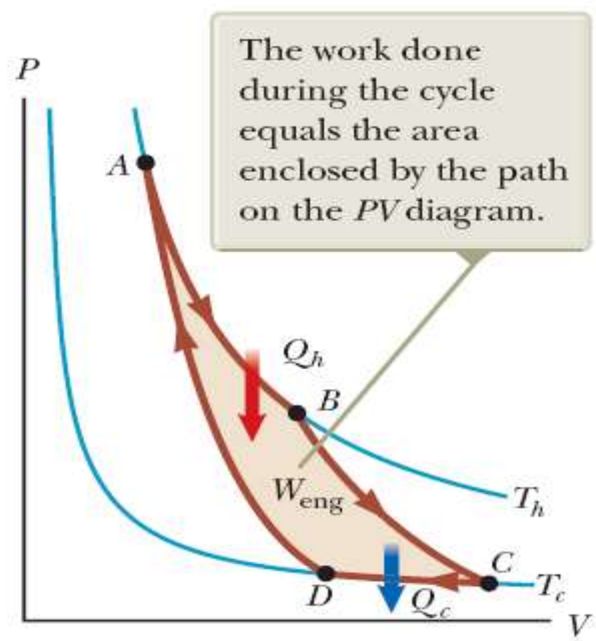
<u>Solution to Question 5</u>

a) Describe in details the four processes of Carnot cycle. Then drive an expression for the efficiency of Carnot engine if the working substance is an ideal gas.

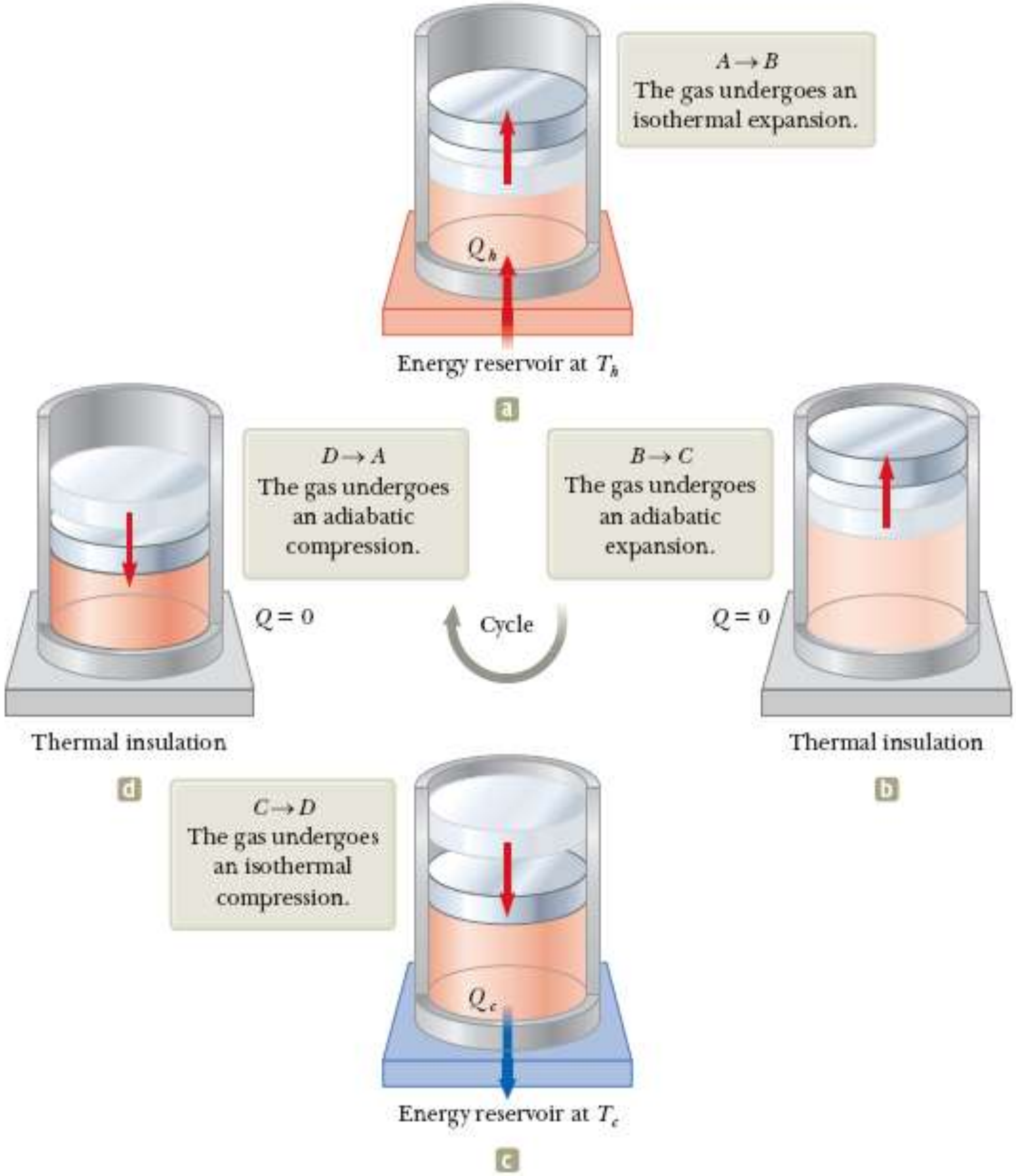
<u>Answer</u>

The Carnot cycle consists of four processes:

- Isothermal expansion,
- Adiabatic expansion,
- Isothermal compression, and
- Adiabatic compression.



Consider Carnot engine consisting of a cylinder fitted with a piston.



Step (1): A ⟶ B Isothermal expansion.

The cylinder is placed on a heat reservoir at temperature T_H.

The gas changes its state (p_A, V_A, T_H) ⟶ (p_B, V_B, T_H)

The gas absorbs heat energy Q_H.

The change in internal energy $\Delta U_{AB} = 0$.

The work done $W = nRT_H \ln(V_B/V_A)$

Step (2): $B \longrightarrow C$ Adiabatic expansion

The cylinder is isolated from its surroundings, so that $Q_{BC} = 0$.
The temperature of the gas decreases from T_H to T_C.

The gas changes its state $(p_B, V_B, T_H) \longrightarrow (p_C, V_C, T_C)$

From the first law of thermodynamics:

The work done $W_{BC} = \Delta U_{Bc} = nC_V (T_C - T_H)$

Step (3): $C \longrightarrow D$ Isothermal compression

The cylinder is placed on a heat reservoir at temperature T_C.

The gas changes its state $(p_C, V_C, T_C) \longrightarrow (p_D, V_D, T_C)$

The gas expells heat energy Q_C.

The change in internal energy $\Delta U_{CD} = 0$.

The work done $W = nRT_C \ln(V_D/V_C)$

Step (4): $D \longrightarrow A$ Adiabatic compression.

The cylinder is isolated from its surroundings, so that $Q_{DA} = 0$.
The temperature of the gas increases from T_C to T_H.

The gas changes its state $(p_D, V_D, T_C) \longrightarrow (p_A, V_A, T_H)$

From the first law of thermodynamics:

The work done $W_{DA} = \Delta U_{DA} = nC_V (T_H - T_C)$

In the whole cycle,

$$\Delta U_{cycle} = 0$$

$W = Q_H - Q_C$

The efficiency $e = W/Q_H = (Q_H - Q_C)/Q_H = 1 - Q_C/Q_H$

b) Choose the correct answer and **justify** your results

Answer:

(1) A sinusoidal wave y(x,t) = 1.75 sin (0.4π x - 280π t) where x any in meters and t in seconds. What is position of the peak at time t = 0.001 s?

(a) 1.25 m **(b) 1.92 m** (c) 2.65 m (d) 3.35 m

The maximum displacement

$$\text{Sin}(0.4\pi\, x - 280\pi\, t) = 1$$

$$(0.4\pi\, x - 280\pi\, t) = (m + ½)\, \pi$$

For the first peak, m = 0:

$$0.4\, x - 280\, t = 1/2$$

At t = 0.001 s

$$0.4\, x = 0.5 + 0.280$$

$$x = 1.25 + 0.7 = 1.95 \text{ m}$$

For m = 1, x =

(2) A sound source is at 530 m away from an observer who hears a sound level of 114.8 dB. What is the intensity heard by another observer at 900 m away from the sound source.

(a) 0.104 W/m² (b) 0.3 W/m² (c) 0.401 W/m² (d) not stated

$$I_2/I_1 = r_1^2/r_2^2$$

$$= (530/900)^2 = 0.3468$$

$\beta_2 - \beta_1 = 10 \log(I_2/I_1)$

$= 10 \log 0.3468$

$= - 4.6 \text{ dB}$

$\beta_2 = 114.8 - 4.6 = 110.2 \text{ dB}$

$\beta_2 = 10 \log (I_2/I_o)$

$I_2/I_o = 10^{11.02} \text{ W/m}^2$

$I_2 = 10^{11.02}\text{x}10^{-12} = 10^{-0.98}$

$= 0.1047 \text{ W/m}^2$

(3) A train passes a standing passenger at a constant speed of 40 m/s. The train horn is sounded at a frequency of 320 Hz. What wavelength is detected by the passenger?
(a) 1.5 m (b) 1.07 m (c) 1.02m **(d) 1.2 m**

$\lambda' = \lambda + v_s/f_o$

$= v/f_o + v_s/f_o$

$= (v + v_s)/f_o$

$= (343 + 40)/320$

$= 1.2 \text{ m}$

(4) A Young double slit interference experiment is carried out with a pair of slits separated by 0.03 mm. The slits are 1.2 m from a screen. The second order maximum is measured to be 4.5 cm from the center line. What is the wavelength of lighjt?
(a) 562.5 nm (b) 654.8 nm (c) 347.6 nm (d) not stated

$y = m\lambda L/d$

$\lambda = yd/mL$

$= (4.5\text{x}10^{-2})(0.03\text{x}10^{-3})/(2)(1.2)$

$= 5.625 \times 10^{-7}$ m = 562.5 nm

(5) A diffraction grating has 10000 lines per centimeter. What is the angle of first order maximum if light of wavelength 600 nm illuminates the grating?

(a) 45.78^o **<u>(b) 36.86^o</u>** (c) 23.58^o (d) not stated

$d = 1 \times 10^{-2}/10000 = 10^{-6}$ m

$d \sin\theta = m\lambda$

$\sin\theta = m\lambda/d$

$= 600 \times 10^{-9}/10^{-6}$

$= 0.6$

$\Theta = 36.87^o$

(6) What is work done by a constant pressure process at 3.324×10^4 Pa to compress a gas from 0.1 m^3 to 0.02 m^3.

<u>(a) - 2659 J</u> (b) - 6783 J (c) - 9543 J (d) not stated

$W = p(V_f - V_i)$

$= (3.324 \times 10^4)(0.02 - 0.1)$

$= - 2659.2$ J

(7) A Carnot engine has an output power of 10 W. The engine operates between 20^oC and 500^oC. What is the thermal energy lost in one hour?

<u>(a) 21990 J</u> (b) 57990 J (c) 36990 J (d) 99034 J

$e = 1 - T_C/T_H$

$= 1 - (293/773)$

$= 0.62$

$W = e\,Q_H$

$Q_H = W/e = 10/0.62 = 16.13$ W

$Q_C = Q_H - W$

$= 16.13 - 10 = 6.13$ W

In one hour, $Q_C = 6.13 \times 60 \times 60 = 22068$ J

(8) A bar of gold (Au) is in thermal contact with a bar of silver (Ag) of the same length and area. One end of the compound bar is maintained at 80.0°C, and the opposite end is at 30.0°C. When the energy transfer reaches steady state, what is the temperature at the junction?[k_{Au} =314 W/m°C, k_{Ag} =427W/m°C]

(a) 51.2°C (b) 34.8°C (c) 76.2°C (d) 21.5 °C

At steady state:

$$(Q/\Delta t)_{Au} = (Q/\Delta t)_{Ag}$$

$$K_{Au} A (T_H - T)/L = K_{Ag} A (T - T_C)/L$$

$$T = (k_{Au} T_H + k_{Ag} T_C)/(k_{Au} + k_{Ag})$$

$$= (314*353 + 427*303)/(314 + 427)$$

$$= 324.2 \text{ K}$$

$$= 51.2^{\circ}\text{C}$$