Benha University **College of Engineering at Banha**
Department of Mechanical Eng.

Subject: **Fluid Mechanics**

Model Answer of the Final Exam **Date: 1/6/2013**

اجابة امتحان ميكانيكا الموائع م 1112 السنة الأولى ميكانيكا

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- **1-a** Recall from thermo class, that a **system** is defined as a volume of mass of fixed identity.
- **Conservation of mass** states that the mass of a system is constant.

This can be written as the following equation:

$$\frac{dm_{sys}}{dt} = 0$$

Conservation of linear momentum

which is a restatement of Newton's Second Law.

Newton's Second Law

- In equation form this is written as: $\Sigma \underline{F}_{sys} = \frac{d}{dt}(m\underline{V})_{sys}$

Where m$\underline{V}$ = the linear momentum of the system.

Conservation of Energy

- For this, use the First Law of Thermodynamics in rate form to obtain the following equation:

$$\frac{dE_{sys}}{dt} = \dot{Q}_{sys} - \dot{W}_{sys}$$

- Where E = the total energy of the system. In the above equation

$$\frac{dE_{sys}}{dt}$$

is the rate of change of system energy.

- $\dot{Q}_{sys}$ is the rate of heat added ***to*** the system $\left(\frac{\delta Q}{dt}\right)$
- $\dot{W}_{sys}$ is the rate of work done ***by*** the system $\left(\frac{\delta W}{dt}\right)$.

Because work is done by the system, the negative sign is in the equation for the first law of thermodynamics.

- Now, these conservation laws must always hold ***<u>for a system</u>***.

<u>Conservation of Angular Momentum</u>

We will have time to study this

1-b

The velocity profile is linear with radius. Additionally, later a discussion on relationship between velocity at interface to solid also referred as the (no) slip condition will be provided. This assumption is good for most cases with very few exceptions. It will be assumed that the velocity at the interface is zero. Thus, the boundary condition is

$U(r = R) = 0$ and $U(r = 0) = U_{max}$ Therefore the velocity profile is

$$U(r) = U_{max}\left(1 - \frac{r}{R}\right) \quad \text{(i)}$$

Where R is radius and r is the working radius (for the integration). The magical averaged velocity is obtained using the equation

. For which

$$\int_0^R U_{max}\left(1 - \frac{r}{R}\right)2\pi r\,dr = U_{ave}\pi R^2$$

The integration of the equation gives

$$U_{max}\pi\frac{R^2}{3} = U_{ave}\pi R^2 \Rightarrow U_{ave} = \frac{U_{max}}{3} \quad \text{(ii)}$$

Calculating the momentum flux correction factor

$$\beta = \frac{1}{A}\int\left(\frac{U}{U_{ave}}\right)^2 dA = \frac{1}{\pi R^2}\int_0^R 3^2\left(1 - \frac{r}{R}\right)^2 2\pi r\,dr$$

$$\beta = \frac{9\times 2\pi}{\pi R^2}\int_0^R\left(1 - \frac{2r}{R} + \frac{r^2}{R^2}\right)r\,dr$$

$$\beta = \frac{18\pi}{\pi R^2}\left(\frac{r^2}{2} - \frac{2r^3}{3R} + \frac{r^4}{4R^2}\right)_0^R = \frac{18\pi}{\pi R^2}\left(\frac{R^2}{12}\right) = 1.5 \quad \text{(iii)}$$

2-a)

The chosen control volume is shown in Figure1. First, the velocity has to be found. This situation is a steady state for constant density. Then

$$U_1 A_1 = U_2 A_2$$

and after rearrangement, the exit velocity is

$$U_2 = \frac{A_1}{A_2}U_1 = \frac{0.0005}{0.0001}\times 5 = 25 m/\sec$$

The momentum equation is applicable but should be transformed into the *z* direction which is

$$\sum F_z + \int_{c.v.} \boldsymbol{g}\cdot\hat{k}\,\rho\, dV + \int_{c.v.} \boldsymbol{P}\cos\theta_z\, dA + \int_{c.v.} \boldsymbol{\tau}_z\, dA = \overbrace{\frac{t}{dt}\int_{c.v.} \rho \boldsymbol{U}_z\, dV}^{=0} + \int_{c.v.} \rho \boldsymbol{U}_z \cdot \boldsymbol{U}_{rn} dA$$

The control volume does not cross any solid body (or surface) there is no external forces. Hence,

$$\overbrace{\sum F_z}^{=0} + \int_{c.v.} \boldsymbol{g}\cdot\hat{k}\,\rho\, dV + \overbrace{\overbrace{\int_{c.v.} \boldsymbol{P}\cos\theta_z\, dA}^{\text{liquid surface}} + \overbrace{\int_{c.v.} \boldsymbol{P}\cos\theta_z\, dA}^{\text{solid surface}}}^{\text{forces on the nozzle } F_{nozzle}} + \int_{c.v.} \boldsymbol{\tau}_z\, dA = \int_{c.v.} \rho \boldsymbol{U}_z \cdot \boldsymbol{U}_{rn} dA$$

All the forces that act on the nozzle are combined as

$$\sum F_{nozzle} + \int_{c.v.} \boldsymbol{g}\cdot\hat{k}\,\rho\, dV + \int_{c.v.} \boldsymbol{P}\cos\theta_z\, dA = \int_{c.v.} \rho \boldsymbol{U}_z \cdot \boldsymbol{U}_{rn} dA$$

The second term or the body force which acts through the center of the nozzle is

$$\boldsymbol{F}_b = -\int_{c.v.} \boldsymbol{g}\cdot\hat{n}\,\rho\, dV = -g\,\rho V_{nozzle}$$

Notice that in the results the gravity is not bold since only the magnitude is used. The part of the pressure which act on the nozzle in the *z* direction is

$$-\int_{c.v.} P dA = \int_1 P dA - \int_2 P dA = PA|_1 - PA|_2$$

The last term in the equation is

$$\int_{c.v.} \rho \boldsymbol{U}_z \cdot \boldsymbol{U}_{rn} dA = \int_{A_2} U_2 (U_2)\, dA - \int_{A_1} U_1 (U_1)\, dA$$

Which results in

$$\int_{c.v.} \rho \boldsymbol{U}_z \cdot \boldsymbol{U}_{rn} dA = \rho\, \left({U_2}^2 A_2 - {U_1}^2 A_1\right)$$

Combining all transform equation into

$$F_z = -g\,\rho V_{nozzle} + PA|_2 - PA|_1 + \rho\, \left({U_2}^2 A_2 - {U_1}^2 A_1\right)$$

$$F_z = -9.81\times1000\times0.0015 + 10^5\times0.0001 - 3\times10^5\times0.0005 + 10^3\times25(25\times0.0001 - 0.0005)$$

$$F_z = -14.715 + 10 - 150 + 50 = -104.715N$$

2-b) For continuity, $Q_3 = Q_1 - Q_2 = 120$ m3/hr. Establish the velocities at each port:

$$V_1 = \frac{Q_1}{A_1} = \frac{220/3600}{\pi(0.045)^2} = 9.61\ \frac{m}{s};\quad V_2 = \frac{100/3600}{\pi(0.035)^2} = 7.22\ \frac{m}{s};\quad V_3 = \frac{120/3600}{\pi(0.02)^2} = 26.5\ \frac{m}{s}$$

With gravity and heat transfer and internal energy neglected, the energy equation becomes

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_3\left(\frac{p_3}{\rho_3} + \frac{V_3^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho_2} + \frac{V_2^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho_1} + \frac{V_1^2}{2}\right),$$

$$\text{or:} \quad -\dot{W}_s/\rho = \frac{100}{3600}\left[\frac{225000}{998} + \frac{(7.22)^2}{2}\right] + \frac{120}{3600}\left[\frac{265000}{998} + \frac{(26.5)^2}{2}\right]$$

$$+ \frac{220}{3600}\left[\frac{150000}{998} + \frac{(9.61)^2}{2}\right]$$

Solve for the shaft work: $\dot{W}_s$ = 998(−6.99 −20.56 +12.00) H =**15500 W** *Ans.*
(negative denotes work done *on* the fluid)

3-a)
For a CV surrounding the tank, with *unsteady* flow, the energy equation is

$$\frac{d}{dt}\left(\int e\rho d\upsilon\right) - \dot{m}_{in}\left(\hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right) = \dot{Q} - \dot{W}_{shaft} = 0, \quad \text{neglect } V^2/2 \text{ and } gz$$

$$\text{Rewrite as} \quad \frac{d}{dt}(\rho\upsilon c_v T) \approx \dot{m}_{in} c_p T_{in} = \rho\upsilon c_v \frac{dT}{dt} + c_v T \upsilon \frac{d\rho}{dt}$$

where 〉 and T are the instantaneous conditions inside the tank. The CV mass flow gives

$$\frac{d}{dt}\left(\int \rho d\upsilon\right) - \dot{m}_{in} = 0, \quad \text{or:} \quad \upsilon\frac{d\rho}{dt} = \dot{m}_{in}$$

Combine these two to eliminate ⌡(d〉/dt) and use the given data for air:

$$\left.\frac{dT}{dt}\right|_{tank} = \frac{\dot{m}(c_p - c_v)T}{\rho\upsilon c_v} = \frac{(0.013)(1005-718)(293)}{\left[\frac{200000}{287(293)}\right](0.2\ m^3)(718)} \approx \mathbf{3.2}\ \frac{\mathbf{°C}}{\mathbf{s}} \quad \textit{Ans.}$$

3-b

i) This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the **boundary layer**.

Boundary layer is the region near a solid where the fluid motion is affected by the solid boundary.

ii) Once the boundary layer has reached the centre of the pipe the flow is said to be **fully developed**. (Note that at this point the whole of the fluid is now affected by the boundary friction.)

At a finite distance from the entrance, the boundary layers merge and the inviscid core disappears. The flow is then entirely viscous, and the axial velocity adjusts slightly further until at $x = L_e$ it no longer changes with x and is said to be fully developed, $v = v(r)$ only.

iii) The length of pipe before fully developed flow is achieved is different for the two types of flow. The length is known as the **entry length**.

The entrance length $\mathbf{L_e}$ is estimated for laminar flow to be :
$L_e/D = 0.06\ Re_D$ for laminar

$L_e/D = 4.4\ Re_D^{1/6}$ for turbulent flow
Where L_e is the entrance length; and
Re_D is the Reynolds number based on Diameter

4-a) The flow rate per unit width of the area $Q = U_o z_0 * 1 = 8 \times 4 \times 1 = 32 cm^3/\sec$

$$Q = \int_0^{z_o} az(z_0 - z)dz = a(\frac{z_0^3}{2} - \frac{z_0^3}{3}) = a\frac{z_0^3}{6}$$

$$\therefore a\frac{4^3}{6} = 32 \Rightarrow a = \frac{6}{2} = 3$$

U_{max} at the middle where $z = \frac{z_o}{2} = 2cm \Rightarrow u_{\max} = 3 \times 2(4-2)$ =12 cm/sec (i)

The shear stress $\tau = \mu\frac{du}{dz}$ at the wall i.e z=0 $\therefore \tau = \mu\frac{du}{dz} = \mu a z_0$ =0.29x3x4=3.48 \n/m^2

The skin friction coefficient

$$C_F = \frac{\tau}{\frac{1}{2}\rho U_o^2} = \frac{3.48}{0.5 \times 891 \times (0.08)^2} = 1.2205 \qquad \text{(ii)}$$

$$\tau dx = dp \times z_o \Rightarrow \frac{dp}{dx} = \frac{\tau}{z_o} = -87 Pa/m \qquad \text{(iii)}$$

4-b)

The displacement thickness is given by

$$\delta^* = \int_0^\infty \left(1 - \frac{v_x}{U}\right)dy = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right)dy$$

$$= \delta - \frac{7\delta}{8} = \frac{\delta}{8} \cong 0.125\ \delta$$

and the momentum thickness is given by

$$\theta = \int_0^\infty \frac{v_x}{U}\left(1 - \frac{v_x}{U}\right)dy = \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right)dy$$

$$= \frac{7\delta}{8} - \frac{7\delta}{9} = \delta\left(\frac{7}{8} - \frac{7}{9}\right) = \frac{7\delta}{72} \cong 0.0972\ \delta$$

Thus, the shape factor is

$$H = \frac{\delta^*}{\theta} = \frac{\frac{\delta}{8}}{\frac{7\delta}{72}} = \frac{9}{7} \cong 1.29$$

5-a) Apply Bernoulli to each pipe separately. For pipe 1:

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + 0.5\frac{u_1^2}{2g} + \frac{4flu_1^2}{2gd_1} + 1.0\frac{u_1^2}{2g}$$

p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A - z_B = \left(0.5 + \frac{4fl}{d_1} + 1.0\right)\frac{u_1^2}{2g}$$

$$10 = \left(1.0 + \frac{4\times 0.008\times 100}{0.05}\right)\frac{u_1^2}{2\times 9.81}$$

$$u_1 = 1.731\, m/s$$

And flow rate is given by:

$$Q_1 = u_1\frac{\pi d_1^2}{4} = 0.0034\, m^3/s$$

For pipe 2:

$$\frac{p_A}{\rho g} + \frac{u_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{u_B^2}{2g} + z_B + 0.5\frac{u_2^2}{2g} + \frac{4flu_2^2}{2gd_2} + 1.0\frac{u_2^2}{2g}$$

Again p_A and p_B are atmospheric, and as the reservoir surface move s slowly u_A and u_B are negligible, so

$$z_A - z_B = \left(0.5 + \frac{4fl}{d_2} + 1.0\right)\frac{u_2^2}{2g}$$

$$10 = \left(1.0 + \frac{4\times 0.008\times 100}{0.1}\right)\frac{u_2^2}{2\times 9.81}$$

$$u_2 = 2.42\, m/s$$

And flow rate is given by:

$$Q_2 = u_2\frac{\pi d_2^2}{4} = 0.0190\, m^3/s$$

i) For the function $P = \text{fcn}(D, \rho, V, \Omega, n)$ the appropriate dimensions are $\{P\} = \{ML^2T^{-3}\}$, $\{D\} = \{L\}$, $\{\rho\} = \{ML^{-3}\}$, $\{V\} = \{L/T\}$, $\{\Omega\} = \{T^{-1}\}$, and $\{n\} = \{1\}$. Using (D, ρ, V) as repeating variables, we obtain the desired dimensionless function:

$$\frac{P}{\rho D^2 V^3} = fcn\left(\frac{\Omega D}{V}, n\right) \quad Ans.\ (\text{i})$$

iii) "Geometrically similar" means that n is the same for both windmills. For "dynamic similarity," the advance ratio ($\Omega D/V$) must be the same:

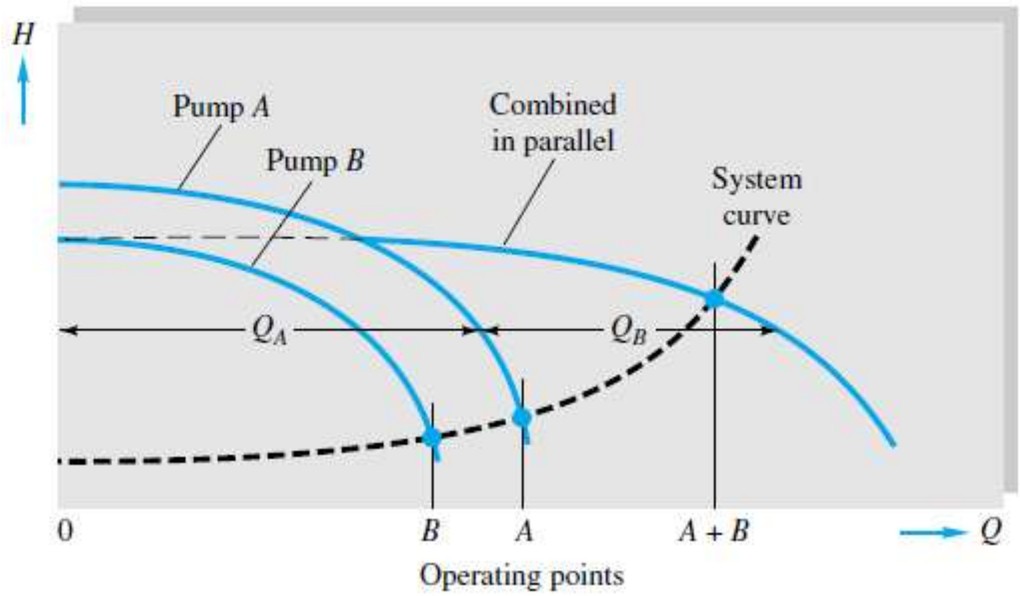
$$\left(\frac{\Omega D}{V}\right)_{model} = \frac{(4800\ r/\text{min})(0.5\ m)}{(40\ m/s)} = 1.0 = \left(\frac{\Omega D}{V}\right)_{proto} = \frac{\Omega_{proto}(5\ m)}{12\ m/s},$$

$$\text{or:} \quad \Omega_{proto} = \mathbf{144\ \frac{rev}{min}} \quad Ans.\ \text{(iii)}$$

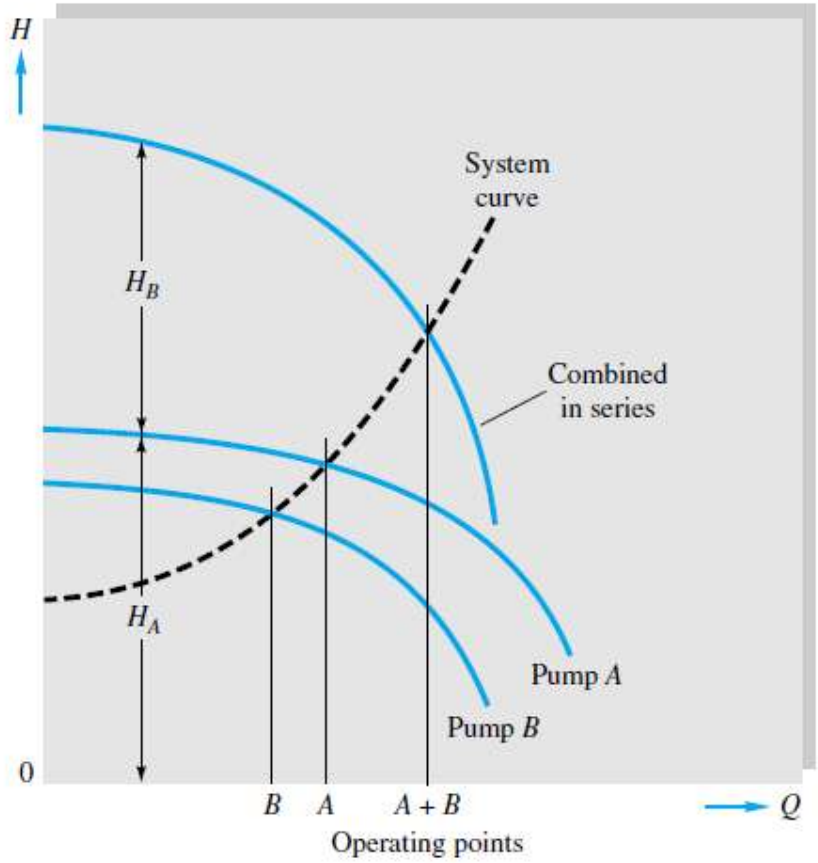
ii) At 2000 m altitude, $\rho = 1.0067$ kg/m^3. At sea level, $\rho = 1.2255$ kg/m^3. Since $\Omega D/V$ and n are the same, it follows that the power coefficients equal for model and prototype:

$$\frac{P}{\rho D^2 V^3} = \frac{2700W}{(1.2255)(0.5)^2(40)^3} = \frac{P_{proto}}{(1.0067)(5)^2(12)^3},$$

$$solve \quad \mathbf{P_{proto}} = 5990\ W \approx \mathbf{6\ kW} \quad Ans.\ \text{(ii)}$$



Performance and operating points of two pumps operating singly and combined in parallel



Performance and operating points of two pumps operating singly and combined in series