Benha University
College of Engineering at Banha
Mechanical Eng. Dept. **4th Year Mechanics**
Subject :Automatic Control **May 19/2013**
Model Answer of the Final Examination
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نموذج اجابة

المادة : التحكم الآلى م 482

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

1-a) The signal flow graph of the sysytem shown in Figure 1. List all loops, , and use Masson rule to find the transfer function of the given system.

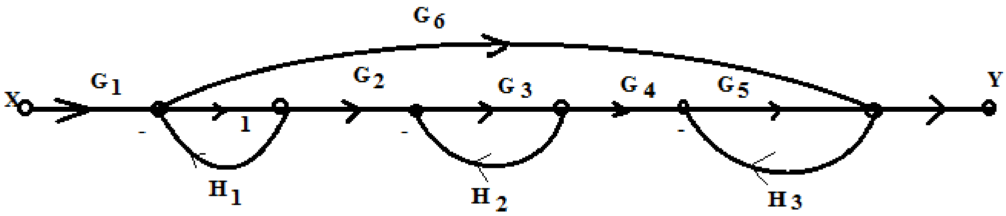


Figure 1

Loops are $L_1 = -H_1$, $L_2 = -G_3H_2$, $L_3 = -G_5H_3$

Paths are $M_1 = G_1\,G_2\,G_3\,G_4\,G_5$, $M_2 = G_1\,G_6$

$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1L_2 + L_1L_3 + L_2L_3) - (L_1L_2L_3)$

$\Delta = 1 + H_1 + G_3H_2 + G_5H_3 + H_1\,G_3H_2 + G_5H_3\,H_1 + G_3H_2G_5H_3 + H_1\,G_3H_2G_5H_3$

$\Delta_1 = 1$

$\Delta_2 = 1 + H_1 + G_3H_2 + G_5H_3 + H_1\,G_3H_2 + G_5H_3\,H_1 + G_3H_2G_5H_3 + H_1\,G_3H_2G_5H_3$

$$TF = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$

1-b)

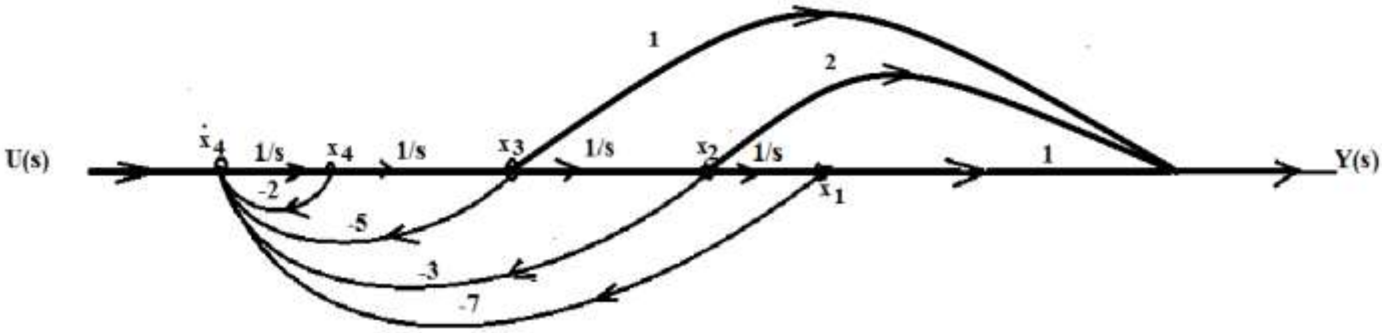
If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 2s + 1}{s^4 + 2s^3 + 5s^2 + 3s + 7}$$

Devide both numenator and denomenator by $\mathbf{s^6}$

we get $$TF = \frac{\frac{1}{s^2} + \frac{2}{s^3} + \frac{1}{s^4}}{1 + \frac{2}{s} + \frac{5}{s^2} + \frac{3}{s^3} + \frac{7}{s^4}}$$

i) signal flow graph represents this system



Ii)Deduce the state space representation of the system., iii

$$\dot{X} = AX + BU = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & -5 & -3 & -7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$\mathbf{Y = CX + DU = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} X + O.U}$$

2-a) . Based on he following graph given in Figure 2, which is the closed-loop step response of a control system.

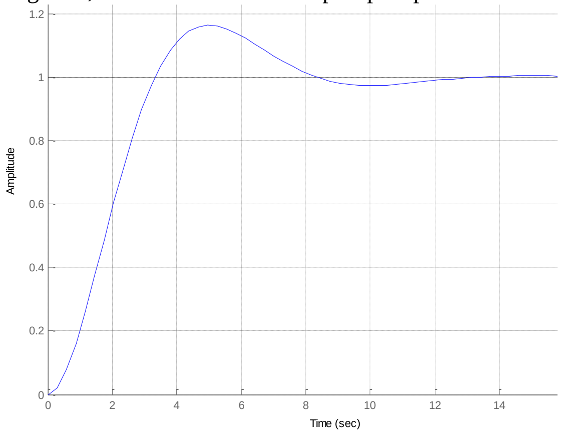


Figure 2

Mp = __16%____, t_p = _5 sec__, t_d = __1.6 sec_, t_r = __3.2 sec__, and t_s = ___12 sec____ for ±2% tolerance.

1- Damping ratio of the pole should be = _0.5_____. So $0 < \zeta < 1$
2- $\omega_n > \omega_d$, ω_d = 0.62 rad/sec
3- $\zeta\,\omega_n$ = 0. 5 , ζ = 0 8 The slope of the open-loop Bode gain plot at very low frequency is _-20___ dB/dec. The low frequency portion has an asymptotic line. The value of this asymptotic line at frequency ω = 1 is equal to -40 dB/dec_____. The Bode phase plot at low frequency will converge to a constant value equal to ___90____ degrees.

1. What is the range of ζ from Figure 1? $0 < \zeta < 1$, $\zeta = 0$ or $\zeta > 1$?
2. If steady state error to unit step input is 0, what is ω_n?
3. If settling time is 8 seconds (wrt 2% criterion), what is ζ? (Note wrt 2% criterion $t_s = 4/\zeta\omega_n$)
4. Write down $G(s)$
5. If you were to apply negative feedback to this $G(s)$ with $H(s) = K$, what do you expect to see in terms of stability when you increase K from 0 to infinity?

2-b)

For the plant $$G_p(s) = \frac{(4-s)}{(s-1)(s+4)}$$

we use a proportional controller $G_c(s) = K$, with $K > 0$.

i) Determine the range of K for which the feedback system is stable.

ii) Draw the Nyqusit plot for $K = 1$.

iii) Design $K > 0$ such that the phase margin is maximized.

Hint: You may use the following identity $\frac{d}{dx}\text{Tan}^{-1}(x) = \frac{1}{1+x^2}$

The characteristic equation is given by 1+KG = 0
This is reduced to $S^2 + (3-k)S + 4(K-1)$
All coefficient should be +ve

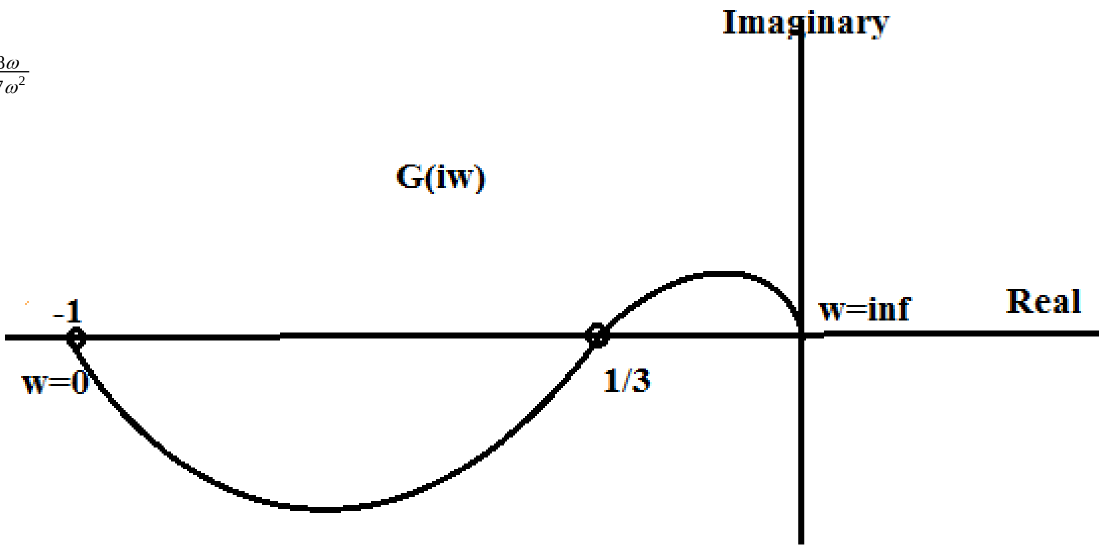
$\therefore 1 \le K \le 3$

ii) $$G_P(i\omega) = \frac{4 - i\omega}{(-4-\omega^2) + 3i\omega} = \frac{-4 + i\omega}{(4+\omega^2) - 3i\omega}$$

$$G_P(i\omega) = \frac{-16 - 7\omega^2 + i(\omega^3 - 8\omega)}{(4+\omega^2)^2 + 9\omega^2}$$

Intersection with real for ω = 0 or $\sqrt{8}$, at -1 and – 0.3333 respectively

$\phi = \tan^{-1} \frac{\omega^3 - 8\omega}{-16 - 7\omega^2}$



The maximum phase will not change with K but the value of $|G(i\omega)|$

Let $\phi = \tan^{-1} x$ for max ϕ, $\frac{d\phi}{d\omega} = \frac{d\phi}{dx}\frac{dx}{d\omega} = \frac{1}{1+x^2}\frac{dx}{d\omega} = 0 \Rightarrow \frac{dx}{d\omega} = 0$

$\frac{dx}{d\omega}$ = 0 for $7\omega^4 + 104\omega^2 - 128 = 0 \Rightarrow \omega^2 = \frac{8}{7}, \Rightarrow \omega = 1.069 s^{-1}$

$G(i1.069) = -0.6533 - 0.1995i$

$|G(i\omega)| = 0.68308 \quad \therefore k = 1.464$ to get tha max phase margin which equal to $\phi = \tan^{-1}\frac{0.1995}{0.6533}$

3-a) Consider a unity gain feedback control system. The plant transfer function is G(s)=1/(s^2+5s+6). Let the controller be of the form C(s) =K(s+z)/(s+p). Design the controller (ie choose K, z, p>0) so that the closed loop system has poles at $-1 \pm j$

The open loop transfer function is given by $C(s)G(s) = \frac{K(s+z)}{(s+p)(s^2+5s+6)}$

The characteristic equation is given by $1 + \frac{K(s+z)}{(s+p)(s^2+5s+6)} = 0$ which is reduced to

$(s+p)(s^2+5s+6) + K(s+z) = 0$

$s^3 + (5+p)s^2 + (6+5p+K)s + (Kz+6p) = 0$

The function is devisible by (s+1-i)(s+1-i) i.e devisible by s^2+2s+2

i.e $s^3 + (5+p)s^2 + (6+5p+K)s + (Kz+6p) = (s^2+2s+2)(s+a) = 0$

$(s^2+2s+2)(s+a) = 0$

$(s^2+2s+2)(s+a) = s^3 + (2+a)s^2 + (2+2a)s + 2a = 0$

Comparing the coefficients

$5 + p = 2 + a \Rightarrow \because p \succ 0 \Rightarrow a \succ 3$

$$6+5(a-3)+k=2+2a$$

$$k=11-3a \Rightarrow a \prec \frac{11}{3}$$

$$kz+6p=2a \rightarrow kz+6(a-3)=2a$$

$$z(11-3a)=18-4a \Rightarrow a \prec \frac{11}{3} \Rightarrow z \succ 0$$

Multiplying the coefficient of s^2 by 2 and subtract the coefficient of s

$$4-3p-k=2 \rightarrow k+3p=2$$

$$0 \prec p \prec \frac{2}{3}, 0 \prec k \prec 2$$

$$kz=2a-6p \Rightarrow 2 \prec kz \prec \frac{22}{3} \;\therefore z \succ 1$$

$$\therefore 0 \prec p \prec \frac{2}{3}, 0 \prec k \prec 2, z \succ 1$$

3-b) For the control system shown in figure 3, sketch the root locus for the following three cases, indicate its direction, where K = 0, where K = ∞, and if they exist, find asymptotes.

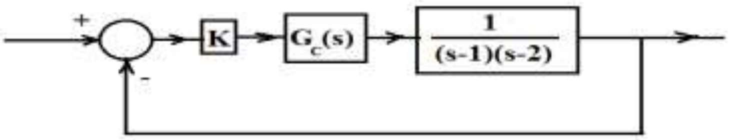
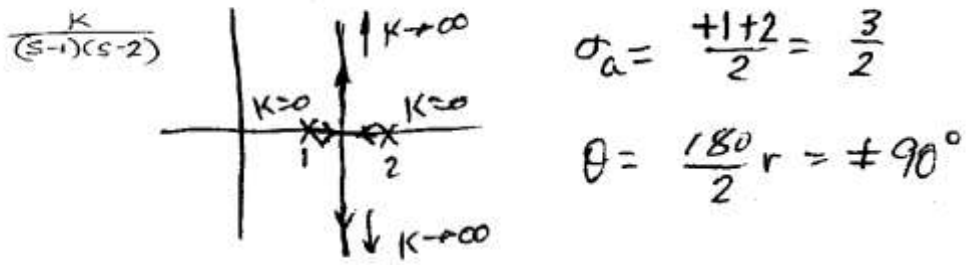


Figure 3

(i) **$G_c(s)$ = 1. , (ii) $G_c(s)$ = s+1 (PD compensation), (iii)$G_c(s)$ =1+1/s (PI compensation).**

For the appropriate choice of compensator, use root locus and Routh Hurwitz techniques to find the range of K for an under damped response

(i)



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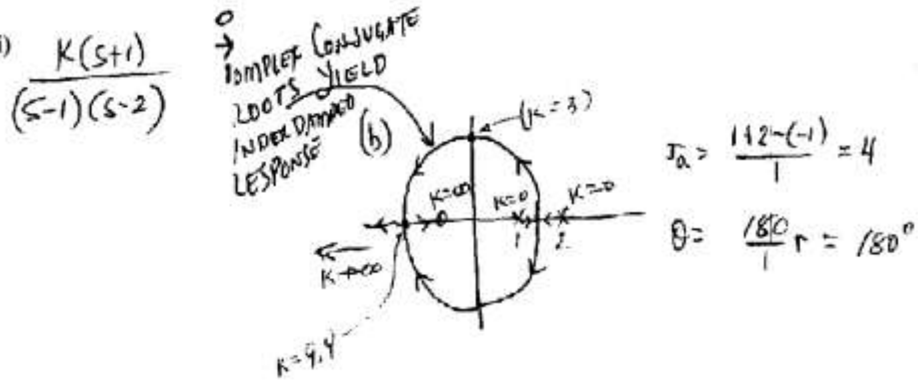
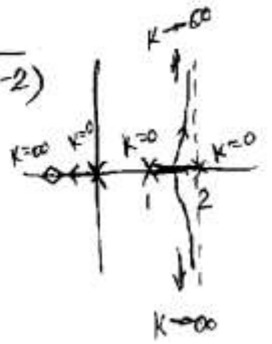
(ii)



(iii) $\frac{K(s+1)}{s(s-1)(s-2)}$



$\sigma_a = \frac{1+2-(-1)}{2} = 2$

$\theta = \frac{180}{2} r = \pm 90^\circ$

Because of the root locus typically approach the asymptotes as in (iii) , but may also lie on the asymptotes as in (i) and (ii)

$1 + \frac{K(s+1)}{(s-1)(s-2)} = 0 \Rightarrow s^2 + (K-3)s + K+2 \Rightarrow K = 3$ FOR MARGINAL STABILITY

$K = -\frac{(s-1)(s-2)}{s+1}$ $\quad \frac{dK}{ds} = 0 \Rightarrow s^2 + 2s - 5 = 0 \Rightarrow s = 1.45, -3.45$

$K\Big|_{s=-3.45} = -\frac{(-3.45-1)(-3.45-2)}{-3.45+1} = 9.9$ $\quad 3 < K < 9.9$ FOR UNDERDAMPED RESPONSE

4-a) Bode Plots of a stable plant $G_P(s)$ are shown in Figure 4 below. Design a proportional controller $G_c(s) = K$, so that the steady state error for a unit step input is as small as possible, and the gain margin of the feedback system is greater or equal to 5 db.

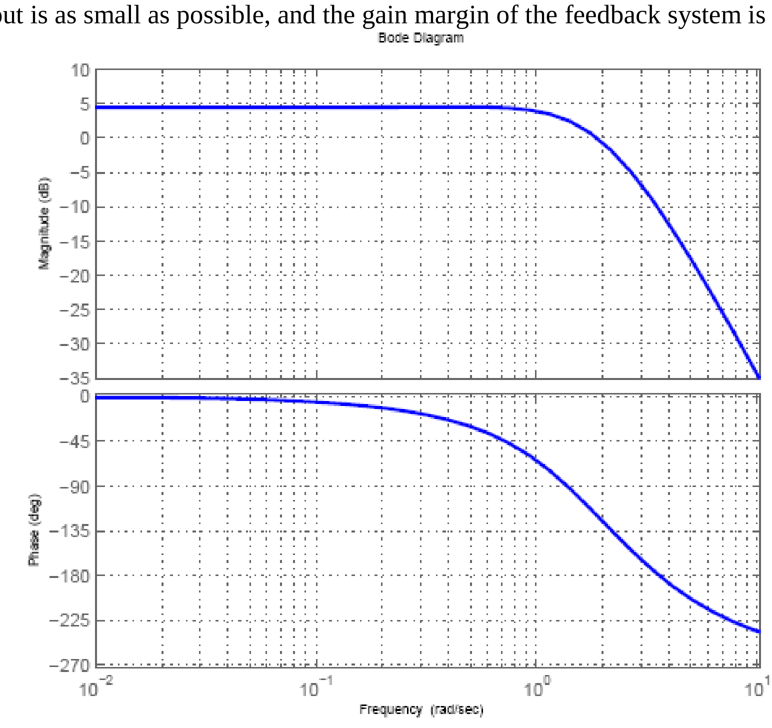
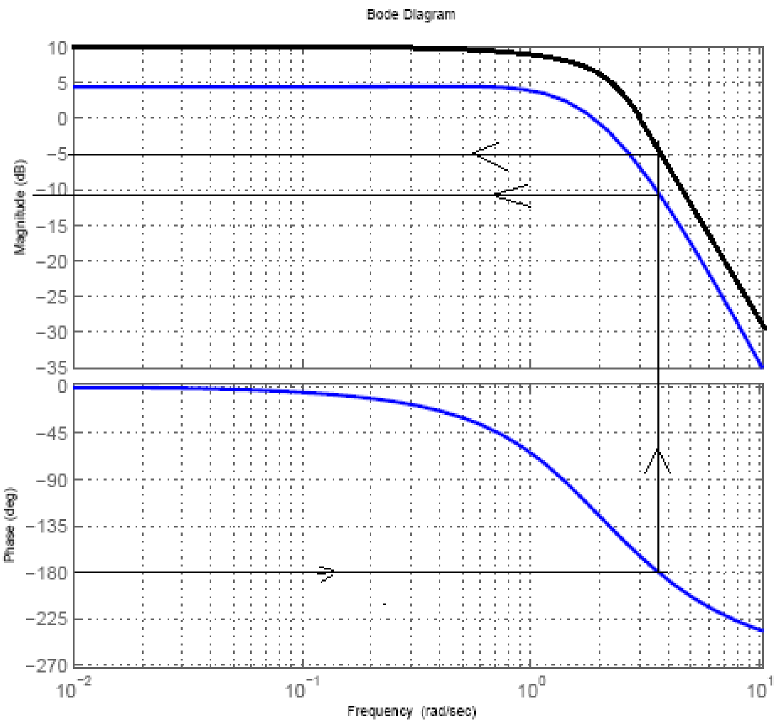


Figure 4

The proportional controller does not chsnge the phase but it does change then gain only

We can shift the Bode plot representing the gain 6db and keep a gain margin of at least 5db as shown in figure below



As ω tends to zero the gain approaches 10

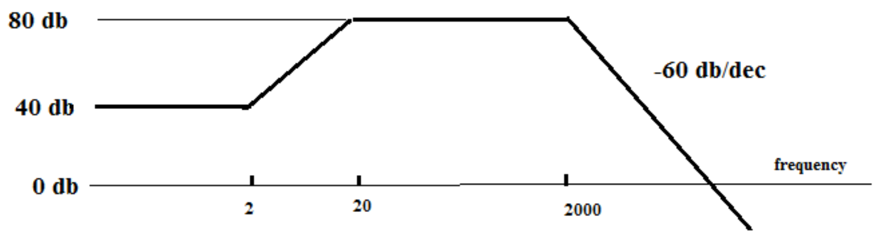
$\therefore 10 = 20\log k_p \Rightarrow k_p = 3.16$

Before the controller $k_p = 10^{0.2} = 1.585$

The steady state error for unit input $e_{ss} = \dfrac{1}{1+k_p}$ as the system is zero type so

The steady state error is reduced from approximately 0.4 to approximately 0.25

4-b) Given the straight line Bode diagram of magnitude in figure 5, find the corresponding transfer function.



Low frequency asymptotes $40db = 20\log x \Rightarrow x = 100$

At frequency $2s^{-1}$ the slope is 40db/dec, there is a factor $\left(\dfrac{s}{2}+1\right)^2$ in the numinator

At frequency $20s^{-1}$ the slope is 0db/dec, there is a factor $\left(\dfrac{s}{20}+1\right)^2$ in the denuminator

At frequency $2000s^{-1}$ the slope is -60db/dec, there is a factor $\left(\dfrac{s}{2000}+1\right)^3$ in the denuminator

$\therefore$ the transfer function G(s) is given by $G(s) = \dfrac{100(\frac{s}{2}+1)^2}{(\frac{s}{20}+1)^2(\frac{s}{2000}+1)^3}$

The system is type ZERO with K_p=100, K_v=0, K_a=0
The steady state error for unit input e_{ss}= $(1/(1+k_p))$ = 1/101
The steady state error for ramp input e_{ss}= $(1/k_v)$ =1/0 = ∞

GOOD LUCK