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نموذج الإجابة امتحان مادة هندسة التحكم ك352 تخلفات مايو 2013
مدرس بالقسم شوقي حامد عرفه

Benha University, Faculty of Engineering	**Elect.Eng.Depart. Third Year 2013**
Subject: Control Engineering (E352)	**Time: 3-hours تخلفات**

Answer

Q1 a- Define: ω_n , ω_d , ω_r , ω_B , ω_c , ω_g , ω_p , M_r, η, G_m, γ_m? (15 marks)

-Natural frequency ω_n rad/sec**:** it is the natural frequency depends on the natural of the system parameters.

- Under damped natural frequency ω_d rad/sec**:** it is the under damped natural frequency depends on the damping coefficient **η** as it is less than one **η <**1.

-Resonant frequency ω_r rad/sec**:** it is the frequency at which the peak value of the output frequency response for a second order is equal to $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}, \quad \text{for } 0 \le \zeta \le 0.707$$

As ζ approaches zero, M_r approaches infinity

$0 < \zeta \le 0.707$, the resonant frequency ω_r is less than the damped natural frequency

-Cut off frequency ω_B rad/sec**:** it is the frequency at which the magnitude of the output frequency response is equal to $(=\frac{1}{\sqrt{2}})$ *of the low frequency* .

-Corner frequency ω_c rad/sec**:** it is the frequency at which the magnitude of the output frequency response is changed sharply. It may be (0, 1, 1/T, **ω_n)**

-Gain crossover frequency ω_g: it is the frequency at which the magnitude of the output frequency response is equal to one or zero decibel.

$|G(j\,\omega g)H(j\,\omega g)| = 1$ or $|G(j\,\omega g)H(j\,\omega g)| = 0db$

-Phase crossover frequency ω_p: it is the frequency at which the phase of the output frequency response is equal to (-180) degrees.

Imag. $[\,G(j\,\omega_p)H(j\,\omega_p)]=0$ or $\angle G(j\,\omega p)H(j\,\omega p) = -180\text{deg}.$

-Maximum resonant magnitude M_r: it is the peak value of the output frequency response for a second order system $\boldsymbol{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad M_r = |G(j\omega)|_{max} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

-damping coefficient η it depends on the natural of the system parameters. For second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Values of η	System stability	Step-response
0> η	System is unstable	undefined
η =0	System is critically stable	oscillatory
0< η <1	System is stable	Under-damped
0< η =1	System is stable	Critically damped
0<η >1	System is stable	Over damped

-Gain margin G_m: it is reciprocal of the magnitude of the output frequency response at the **Phase crossover frequency ω_p**

$\mathbf{G_m}$=1/[Real of G(j ω_p)H(j ω_p)]= 1/|G(j $\boldsymbol{\omega p}$)H(j $\boldsymbol{\omega p}$)| =K_c/K

G_M=20log G_m db

-Phase margin γ_m: it is the angle of the output frequency response at the **gain crossover** frequency plus 180 degrees.

$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180$ deg.

b- Consider a control system shown in Fig.1 if **G(S) =4/[S(S+2)], H(S) =1**

i-Find the **frequency response** as r(t)=5sinωt **?** ii-Calculate **M_r, ω_r?**

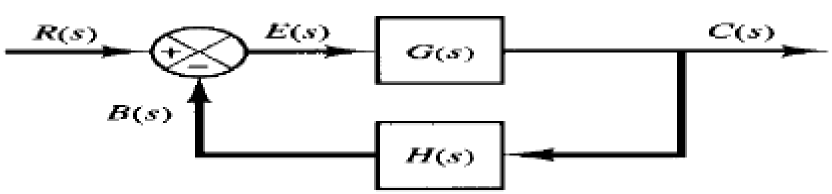


Fig.1

Frequency Response: it means the steady state output of a 1-linear 2-time-invariant 3- stable control system to a sinusoidal input and it is a sinusoidal with phase shift positive or negative and does not depend on the initial conditions.

b-Steps to find frequency Response:

1- the closed loop transfer function =**T(s)=C(S)/R(S)** =

$$C(S) / R(S) = \frac{G(s)}{1+G(S)H(S)} = \frac{\omega_n^2}{S^2+2\eta\omega_n S+\omega_n^2} = \frac{4}{S^2+2S+4},\ \omega_n = \frac{2rad}{sec}\ \zeta = 0.5$$

2-the closed loop frequency transfer function =

$$\mathbf{T\ (j\omega)=C(j\omega)/R(j\omega)} = \frac{4}{(j\omega)^2+2(j\omega)+4} = M\llcorner\Phi = Re+j\ imag$$

$$\boldsymbol{M} = \frac{\mathbf{4}}{\sqrt{(\mathbf{4}-\boldsymbol{\omega}^2)^2+\mathbf{4}\boldsymbol{\omega}^2}}\quad ,\ \boldsymbol{\Phi} = \mathbf{tan^{-1}}[\mathbf{2}\boldsymbol{\omega}\,/(\mathbf{4}-\boldsymbol{\omega}^2)]$$

3-As the input $=r(t) = 5sin\omega t$ then

the response = $C(t) = 5Msin(\omega t + \Phi)$

$$= \frac{\mathbf{20}}{\sqrt{(\mathbf{4}-\boldsymbol{\omega}^2)^2+\mathbf{4}\boldsymbol{\omega}^2}}\ \sin[\omega t + \mathbf{tan^{-1}}[\mathbf{2}\boldsymbol{\omega}\,/(\mathbf{4}-\boldsymbol{\omega}^2)]$$

$$\boldsymbol{M_r} = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\zeta}\sqrt{\mathbf{1}-\boldsymbol{\zeta}^2}} = \frac{\mathbf{1}}{\mathbf{2(0.5)}\sqrt{\mathbf{1}-\mathbf{(0.5)}^2}} = 1.155$$

$$\boldsymbol{\omega_r} = \boldsymbol{\omega_n}\sqrt{\mathbf{1}-\mathbf{2}\boldsymbol{\zeta}^2} = \mathbf{2}\sqrt{\mathbf{1}-\mathbf{2(0.5)}^2} = \mathbf{1.414}\ \boldsymbol{rad/sec}.$$

Q2 (15 marks)

Consider a control system shown in Fig.1 if **G(S) =K/[(S+3)(S+2)], H(S) =1/(S+1)**

a- Sketch the **complete root locus** for positive values of **K**?
b- Find **K** that makes the complex closed loop poles have a damping ratio =**0.6** and **find the closed loop poles** using **the plot**?
c- Find **K** that makes the complex closed loop poles have a damping ratio =**0.6** and **find the closed loop poles analytically**?
d- Write short MATLAB program to solve **a** and solve **b**?

Root locus:

1-the root locus is symmetrical about the real axis in the S-plane

2-the open loop TF=G(s) H(s)= **G(S) H(S) =K/[(S+3)(S+2)(S+1)]**=K/[S^3+6S^2+11s+6]

3-the root locus starts at the pole and ends at the zero or infinity

4-number of root loci= n=number of poles of the open loop TF =3 at [-1,-2,-3]

5-number of zeros= m=0

6-number of asymptotes = n-m=3-0=3

8-center of gravity = $A = \frac{\sum poles - \sum zoles}{n-m} = \frac{-1-2-3}{3} = -2$point of intersection of asymptotes with real axis=

9-angles of asymptotes are = $\Theta = \frac{\pm 180(2R+1)}{n-m} = \pm 60$,±180

10- Points of crossing the imaginary axis as Routh test

Charct.equa=1+G(S)H(S)=0= S^3+6S^2+11s+6+K

S^3	1	11	6+K≥0, [66-6-K]/ 6≥0 then -6≤K≤60,Kc=60 6S^2+6+60=0, S=j ω =$\sqrt{11}$ rad/sec
S^2	6	6+K	
S	[66-6-K]/ 6		
S^0	6+K		

11- break points (break away or break in) at

$$-\frac{dK}{dS} = 0 = \frac{d}{dS}\left[\frac{1}{G(S)H(S)}\right] = \frac{d}{dS}[S^3 + 6S^2 + 11s + 6] = 3S^2 + 12s + 11 = 0$$

S=-2.6 refused, S=-1.4 is a break- away point

12-break angles at [±180(2R+1)/r] where r=number of branches(poles for break away or zeros for break in) R=0,1,----- break angles at [±180]/2=±90

13-there is no angle of departure (complex poles)

14- there is no angle of arrival (complex zeros)

15-sketch the root loci as

16- the damping factor or coefficient ζ is straight line with slope Θ= $\cos^{-1}\zeta$

with respect to the negative real axis in the S-plane. $\Theta = \cos^{-1} 0.6$ =53.13deg. at the test point (intersection point) S_d=-1±j1.4

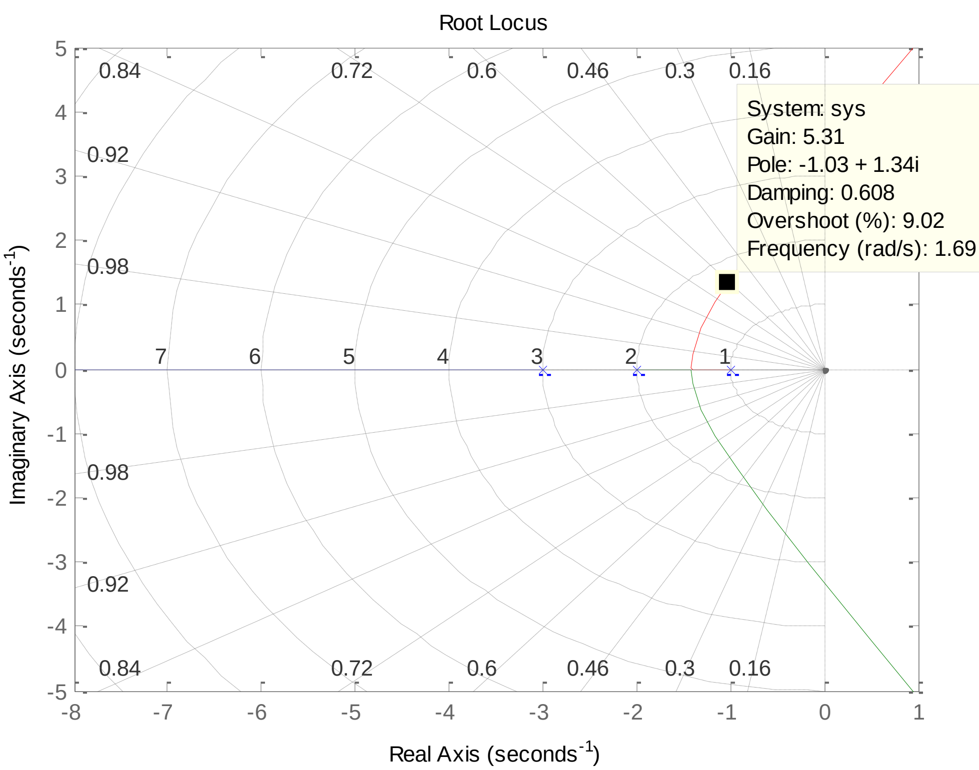
$$\text{angle condition} = \sum_{n=1}^{n=3} [\Theta_{zeros} - \Theta_{poles}] = \pm 180(2R+1) = 90 + 54 + 36 = 180 \text{ deg}$$

$$\text{magnitude condition} = \sum_{n=1}^{n=3} \frac{\|\text{poles}\|}{\|\text{zeros}\|} = K = 1.4 * 1.6 * 2.4 = 5.3$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = \sum_{n=1}^{n=3} closed\ loop\ poles = \text{constant} \ \text{ as } n - m \geq 2$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = -1 - 2 - 3 = -6 = \sum_{n=1}^{n=3} closed\ loop\ poles = 2(-1) \pm j1.4 + p$$

$$\text{then } p = -4 \text{ i.e. closed loop poles are } [-1 \pm j1.4, -4\,]$$



19- To find analytically closed loop poles and K as

$(S^2+2\zeta\omega_n S+\omega_n^2)(S+a)$=characteristic equa. for a third order syst.

Solve $1+G(S)H(S)=0=S^3+6S^2+11s+6+K=(S^2+1.2\omega_n S+\omega_n^2)(S+a)$

$$=S^3+(1.2\omega_n+a)S^2+(1.2\omega_n a+\omega_n^2)S+\omega_n^2 a$$

$1.2\omega_n+a=6$,,,, $1.2\omega_n a+\omega_n^2=11$,,,, $\omega_n^2 a=k+6$

Prog. >>n=[1];d=[1 6 11 6]; rlocus(n,d), grid

Q3 (30marks)

Consider a control system shown in Fig.1 if H(s)=1/(S+2), G(s)=k/(S+1)(s+3),

- a- Prove that the gain margin=6.02db at 3.32rad/sec. and the phase margin=25.4 degrees at 2.35rad/sec. **as K=30**?
- b- Sketch the **polar plot as K=30**?
- c- Sketch the **Bode plot as K=30**?
- d- Find the gain margin and the phase margin using **the plots**?
- e- Write a short MATLAB program to solve b ,c and d?
- f- Write short MATLAB program to Sketch the **Nichols plot**?

1- the open loop TF=G(s) H(s)= **G(S) H(S) =K/[(S+3)(S+2)(S+1)]**$=30/[S^3+6S^2+11s+6]$

2- Find the freq.open loop TF=

$$\mathbf{G(j\omega)H(j\omega)}=\frac{30}{[(j\omega+3)(j\omega+2)(j\omega+1)]}=Me^{j\Phi}=M\angle\Phi=\text{Re}+j\,\text{imag}$$

$$\boldsymbol{M}=\frac{\mathbf{30}}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}\sqrt{9+\omega^2}}\quad,$$

$$\boldsymbol{\Phi}=-\tan^{-1}(\boldsymbol{\omega})-\tan^{-1}(\boldsymbol{\omega}/\mathbf{2})-\tan^{-1}(\boldsymbol{\omega}/\mathbf{3})]$$

$$M=\frac{30}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}\sqrt{9+\omega^2}}=\frac{30}{\sqrt{1+2.35^2}\sqrt{4+2.35^2}\sqrt{9+2.35^2}}=1$$

$$\boldsymbol{M}=\frac{\mathbf{30}}{\sqrt{\mathbf{1}+\boldsymbol{\omega}^2}\sqrt{\mathbf{4}+\boldsymbol{\omega}^2}\sqrt{\mathbf{9}+\boldsymbol{\omega}^2}}=\frac{\mathbf{30}}{\sqrt{\mathbf{1}+\mathbf{2.35}^2}\sqrt{\mathbf{4}+\mathbf{2.35}^2}\sqrt{\mathbf{9}+\mathbf{2.35}^2}}=\mathbf{1}$$

$$\boldsymbol{M}=\frac{\mathbf{30}}{\sqrt{\mathbf{1}+\boldsymbol{\omega}^2}\sqrt{\mathbf{4}+\boldsymbol{\omega}^2}\sqrt{\mathbf{9}+\boldsymbol{\omega}^2}}=\frac{\mathbf{30}}{\sqrt{\mathbf{1}+\mathbf{3.32}^2}\sqrt{\mathbf{4}+\mathbf{3.32}^2}\sqrt{\mathbf{9}+\mathbf{3.32}^2}}=\mathbf{0.5}$$

$$\boldsymbol{\Phi}=-\tan^{-1}(\mathbf{3.32})-\tan^{-1}(\mathbf{3.32}/\mathbf{2})-\tan^{-1}(\mathbf{3.32}/\mathbf{3})]=-180\text{ deg.}$$

$\boldsymbol{\Phi} = -\tan^{-1}(2.35) - \tan^{-1}(2.35/2) - \tan^{-1}(2.35/3)] = -154.6$ **deg.**

$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180\text{ deg.} = 180 - 154.6 = 25.4deg.$

3- Find the table

ω	0	0.1	1	2.35	3.32	5	10	∞
Φ	0	-10.5	-90	-155	-180	-206	-236	-270
M	5	4.97	3	1	0.5	0.2	0.03	0
20logM	14	13.9	9.5	0	6.4	-14.6	-31.1	0
Real **G(jω)H(jω)**	5		0					0
Imag **G(jω)H(jω)**	0		-3		0			0

4- Plot the vector on the $\mathbf{j\omega -}$ **plane** where Φ in degrees as a straight line and determine M on this line
5- Plot the locus of the vector as points from the table
6- Find the gain and the phase margins from the plot

Prog. >>n=[1]; d=[1 6 11 6];

```
>> nyquist(n,d)      >> margin(n,d)      >> nichols(n,d)
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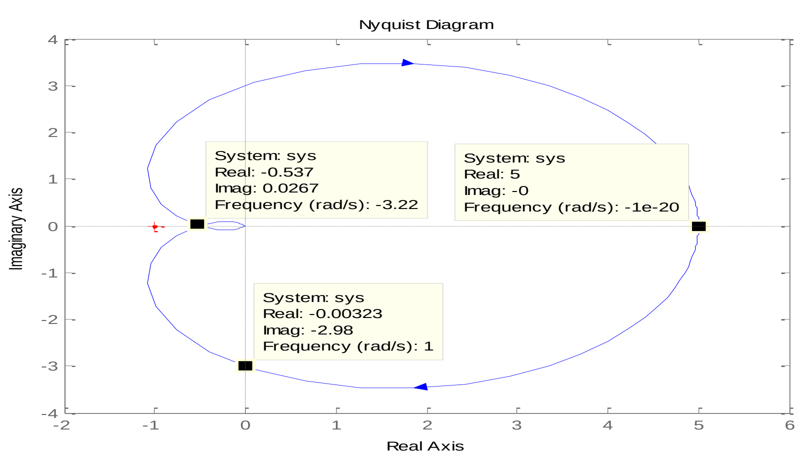
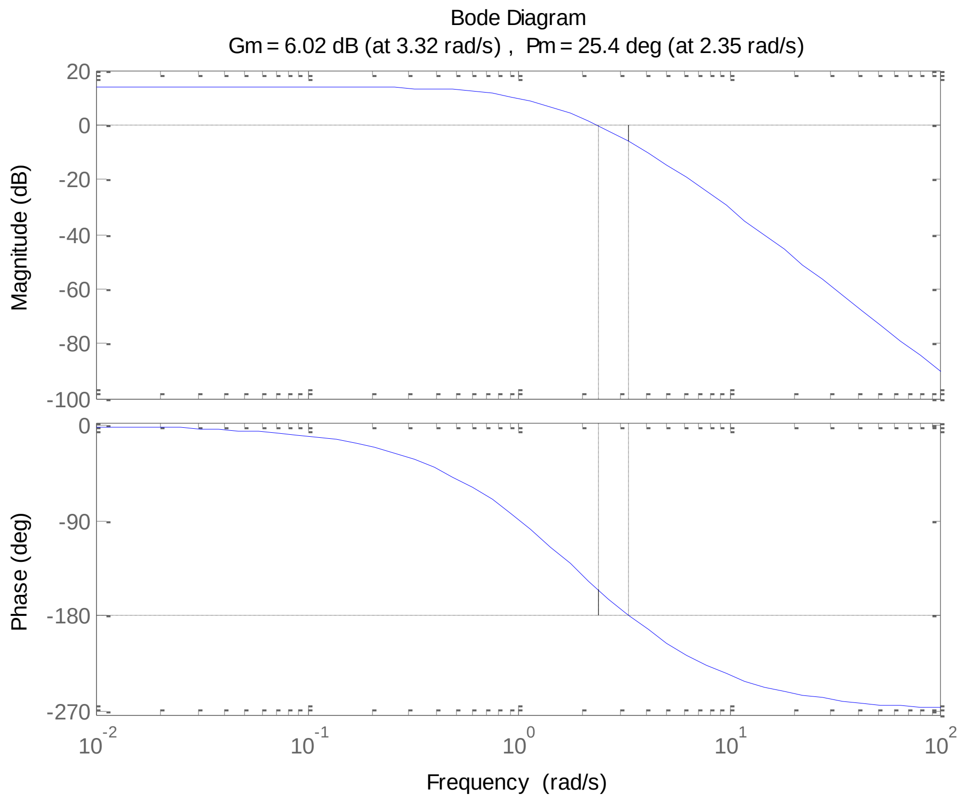
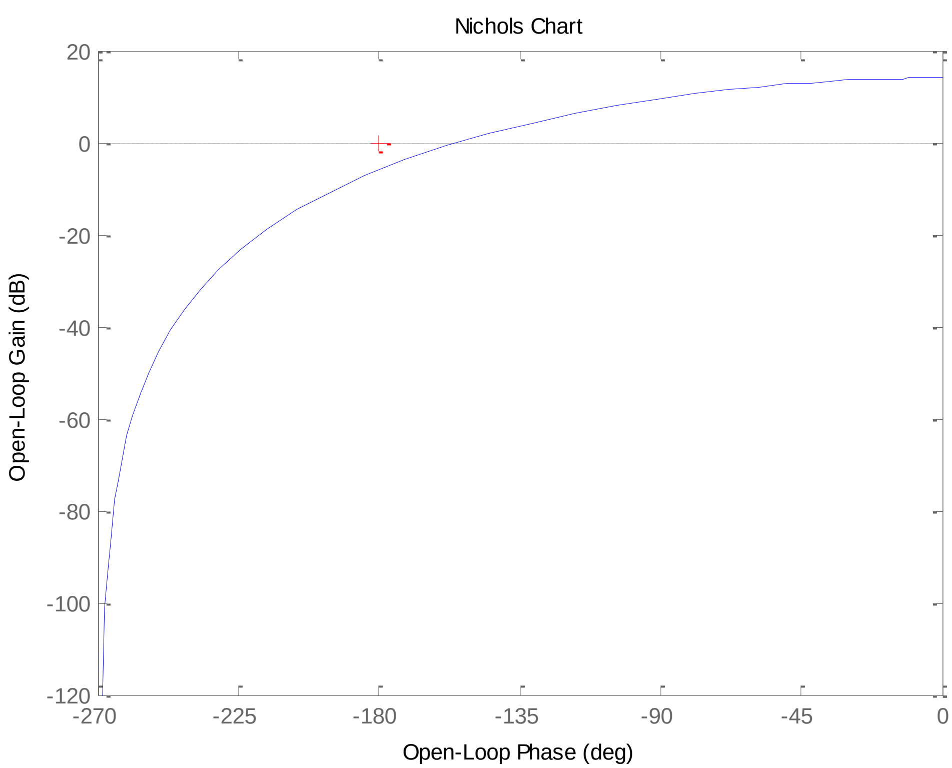



Bode Diagram
Gm = 6.02 dB (at 3.32 rad/s) , Pm = 25.4 deg (at 2.35 rad/s)
Magnitude (dB)
20
0
-20
-40
-60
-80
-100
Phase (deg)
0
-90
-180
-270
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/s)


Nichols Chart
Open-Loop Gain (dB)
20
0
-20
-40
-60
-80
-100
-120
-270
-225
-180
-135
-90
-45
0
Open-Loop Phase (deg)