Benha University
College of Engineering at Banha
Mechanical Eng. Dept.
Subject :Automatic Control

4th Year Mechanics
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Model Answer of Final Examination For external Students

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نموذج اجابة امتحان الطلبة من الخارج

المادة : التحكم الآلى م 482

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

1-a) Find the signal flow graph of the sysytem shown in Figure 1 Use the given variables as nodes. List all loops, , and use Masson rule to find the transfer function of the given system.

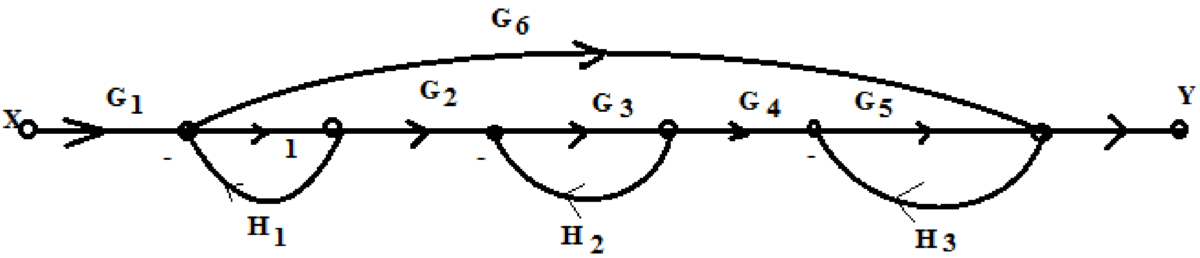


Figure 1

Loops are $L_1 = - H_1$, $L_2 = - G_3H_2$, $L_3 = - G_5H_3$

Paths are $M_1 = G_1\, G_2\, G_3\, G_4\, G_5$, $M_2 = G_1\, G_6$

$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1L_2 + L_1L_3 + L_2L_3) - (L_1L_2L_3)$

$\Delta = 1 + H_1 + G_3H_2 + G_5H_3 + H_1\, G_3H_2 + G_5H_3\, H_1 + G_3H_2G_5H_3 + H_1\, G_3H_2G_5H_3$

$\Delta_1 = 1$

$\Delta_2 = 1 + H_1 + G_3H_2 + G_5H_3 + H_1\, G_3H_2 + G_5H_3\, H_1 + G_3H_2G_5H_3 + H_1\, G_3H_2G_5H_3$

$$TF = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$

1-b)

For the following block diagram (Fig.2), Find:

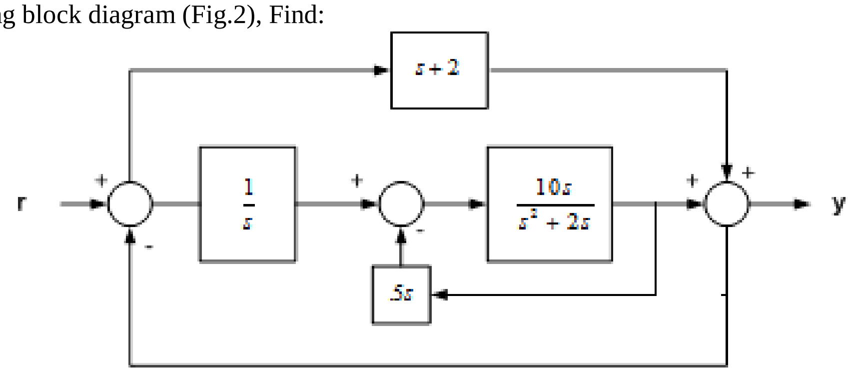
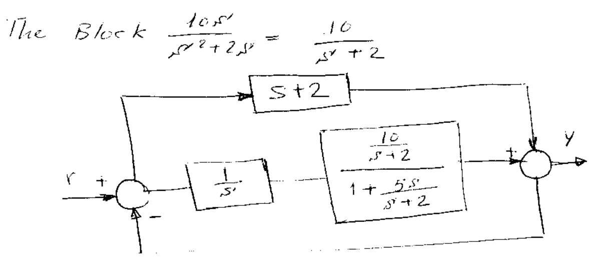
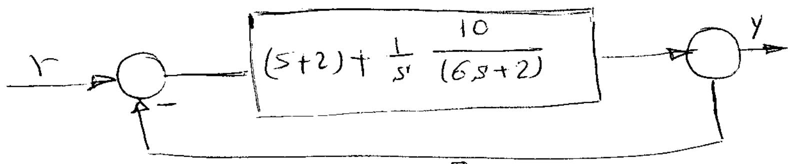


Figure 2



The system is reduced to



(i) $T.F = \dfrac{(s+2) + \dfrac{5}{3s^2+s}}{1 + (s+2) + \dfrac{5}{3s^2+s}}$

$= \dfrac{3s^3 + 7s^2 + 2s + 5}{3s^3 + 10s^2 + 3s + 5}$

If r is unit step, The open loop T.F

$G(s) = (s+2) + \dfrac{1}{s}\left(\dfrac{10}{6s+2}\right)$

(ii) $K_p = \lim_{s \to 0} G(s) = \infty \Rightarrow e_{ss} = \dfrac{1}{1+K_p} = 0$

(iii) $K_v = \lim_{s \to 0} s\,G(s) = 5 \Rightarrow e_{ss} = \dfrac{1}{5} = 20\%$

Construct The Hourwitz array

s^3	3	3
s^2	10	5
s	1.5	0
1	5	0

The system is BIBO stable.

2-a) . Based on the following graph given in Figure 3, which is the closed-loop step response of a control system.

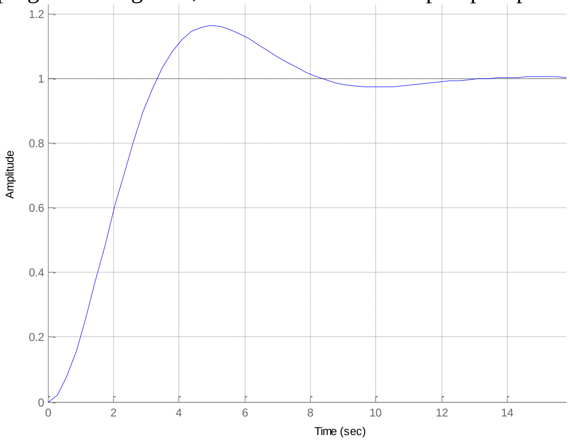


Figure 3

Mp = __16%____, t_p = _5 sec__, t_d = __1.6 sec_, t_r = __3.2 sec__, and t_s = ___12 sec____ for ±2% tolerance.

ii) The dominant pole pair of the system must be at p = __-0.36___ + j_0.62____; and damping ratio of the pole should be = _0.5_____.

iii) If one had performed the open-loop frequency response and obtained the Bode plot for the open-loop system, the Bode plot would have a gain cross over frequency at ω_{gc} = _0.512_____ and a phase margin = 55 degree______.

iv) The slope of the open-loop Bode gain plot at very low frequency is _-20___ dB/dec. The low frequency portion has an asymptotic line. The value of this asymptotic line at frequency ω = 1 is equal to -40 dB/dec_____. The Bode phase plot at low frequency will converge to a constant value equal to ___90____ degrees.

2-b)

For the plant $G_p(s) = \frac{(4-s)}{(s-1)(s+4)}$

we use a proportional controller $G_c(s) = K$, with $K > 0$.

i) Determine the range of K for which the feedback system is stable.

ii) Draw the Nyqusit plot for $K = 1$.

iii) Design $K > 0$ such that the phase margin is maximized.

Hint: You may use the following identity $\frac{d}{dx}\mathrm{Tan}^{-1}(x) = \frac{1}{1+x^2}$

i) The characteristic equation is given by 1+KG = 0

This is reduced to $S^2 + (3-k)S + 4(K-1)$

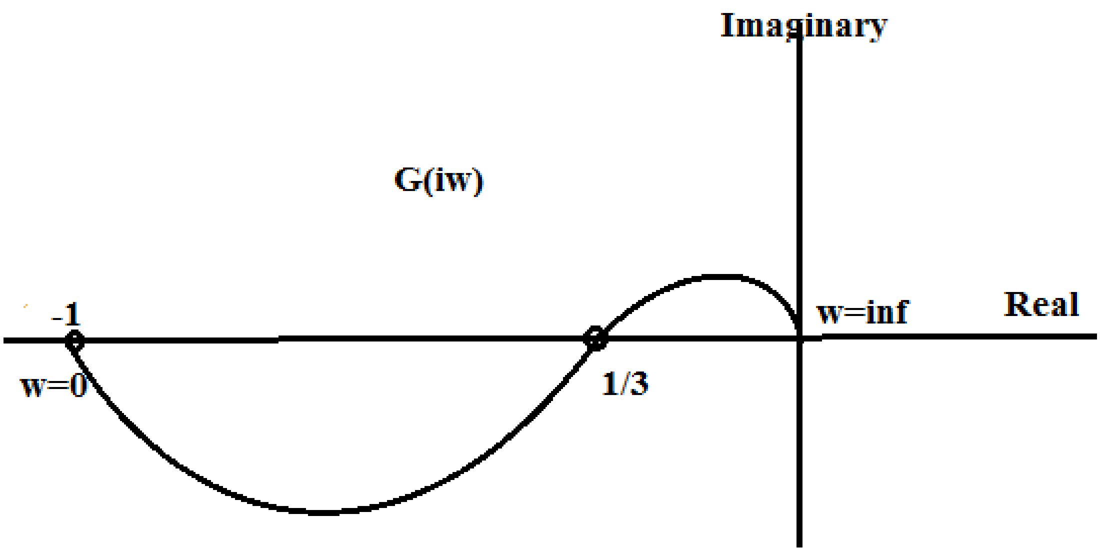
All coefficient should be +ve

$\therefore 1 \le K \le 3$

ii) $G_p(i\omega) = \frac{4-i\omega}{(-4-\omega^2)+3i\omega} = \frac{-4+i\omega}{(4+\omega^2)-3i\omega}$

$$G_P(i\omega) = \frac{-16-7\omega^2 + i(\omega^3 - 8\omega)}{(4+\omega^2)^2 + 9\omega^2}$$

Intersection with real for $\omega = 0$ or $\sqrt{8}$, at -1 and – 0.3333 respectively



$\phi = \tan^{-1} \frac{\omega^3 - 8\omega}{-16-7\omega^2}$

The maximum phase will not change with K but the value of $|G(i\omega)|$

Let $\phi = \tan^{-1} x$ for max ϕ, $\frac{d\phi}{d\omega} = \frac{d\phi}{dx}\frac{dx}{d\omega} = \frac{1}{1+x^2}\frac{dx}{d\omega} = 0 \Rightarrow \frac{dx}{d\omega} = 0$

$\frac{dx}{d\omega}$ = 0 for $7\omega^4 + 104\omega^2 - 128 = 0 \Rightarrow \omega^2 = \frac{8}{7}, \Rightarrow \omega = 1.069 s^{-1}$

$G(i1.069) = -0.6533 - 0.1995i$

$|G(i\omega)| = 0.68308 \quad \therefore k = 1.464$ to get tha max phase margin which equal to $\phi = \tan^{-1}\frac{0.1995}{0.6533}$

3-a) Consider a unity gain feedback control system. The plant transfer function is G(s)=1/(s^2+5s+6). Let the controller be of the form C(s) =K(s+z)/(s+p). Design the controller (ie choose K, z, p>0) so that the closed loop system has poles at $-1 \pm j$

The open loop transfer function is given by $C(s)G(s) = \dfrac{K(s+z)}{(s+p)(s^2+5s+6)}$

The characteristic equation is given by $1 + \dfrac{K(s+z)}{(s+p)(s^2+5s+6)} = 0$ which is reduced to

$(s+p)(s^2+5s+6) + K(s+z) = 0$

$$s^3 + (5+p)s^2 + (6+5p+K)s + (Kz+6p) = 0$$

The function is devisible by (s+1-i)(s+1-i) i.e devisible by s^2+2s+2

i.e $s^3 + (5+p)s^2 + (6+5p+K)s + (Kz+6p) = (s^2+2s+2)(s+a) = 0$

$$(s^2+2s+2)(s+a) = 0$$

$$(s^2+2s+2)(s+a) = s^3 + (2+a)s^2 + (2+2a)s + 2a = 0$$

Comparing the coefficients

$$5 + p = 2 + a \Rightarrow \because p \succ 0 \Rightarrow a \succ 3$$

$$6 + 5(a-3) + k = 2 + 2a$$

$$k = 11 - 3a \Rightarrow a \prec \frac{11}{3}$$

$$kz + 6p = 2a \rightarrow kz + 6(a-3) = 2a$$

$$z(11-3a) = 18 - 4a \Rightarrow a \prec \frac{11}{3} \Rightarrow z \succ 0$$

Multiplying the coefficient of s^2 by 2 and subtract the coefficient of s

$$4 - 3p - k = 2 \rightarrow k + 3p = 2$$

$$0 \prec p \prec \frac{2}{3}, 0 \prec k \prec 2$$

$$kz = 2a - 6p \Rightarrow 2 \prec kz \prec \frac{22}{3} \; \therefore z \succ 1$$

$$\therefore 0 \prec p \prec \frac{2}{3}, 0 \prec k \prec 2, z \succ 1$$

3)

b) Hand sketch the root locus of 1 + KG(s) = 0 as K varies from 0 to +∞, where

$$G(s) = \frac{s+2}{s(s+1)(s+3)^2}$$

→ open loop poles are 0, −1, −3, −3

" " zero are −2

Number of asymptotes are n − m = 3

asymptotes angle = $\dfrac{(2k+1)\pi}{3}$ = 60°, 180°, 300°

Intersection of asymptotes on real axis

$$\alpha = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \frac{-1 - 3 - 3 + 2}{3} = \frac{-5}{3}$$

The characteristic eqn. is

$$1 + KG(s) = 0 \Rightarrow 1 + \frac{k(s+2)}{(s^2+s)(s^2+6s+9)} = 0$$

$$s^4 + 7s^3 + 15s^2 + 9s + Ks + 2K = 0$$

Construct the array.

s^4	1	15	$2K$
s^3	7	$(9+K)$	0
s^2	$\frac{105-9K-9}{7}$	$2K$	
s	$\frac{(96-9K)(9+K)-98K}{96-9K}$	0	
1	$2K$		

$96 - 9K > 0 \Rightarrow K < \frac{96}{9}$

$(96 - 9K)(9+K) - 98K > 0 \Rightarrow$

$864 - 81K - 98K - 9K^2 > 0$

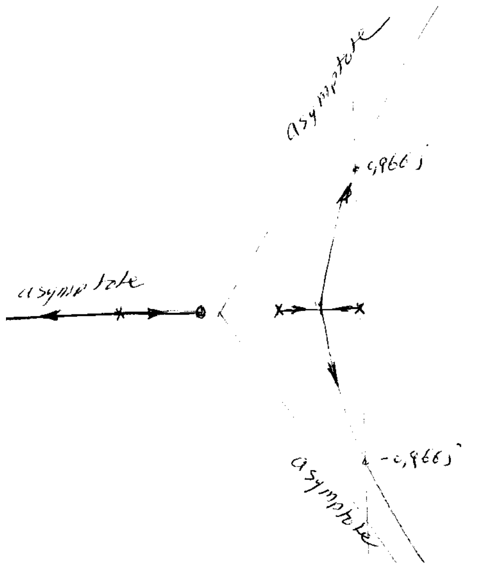
$864 - 179K - 9K^2 > 0 \quad K \leq 4,$

For $K = 4$

The Auxiliary Eqn. is $\frac{60}{7}s^2 + 8 = 0$

$s = \pm\sqrt{\frac{14}{15}}\, j$ points intersect the Imaginary line.

$\approx \pm 0.966j$



The root locus

4-a) Bode Plots of a stable plant $G_P(s)$ are shown in Figure 4 below. Design a proportional controller $G_c(s) = K$, so that the steady state error for a unit step input is as small as possible, and the gain margin of the feedback system is greater or equal to 5 db.

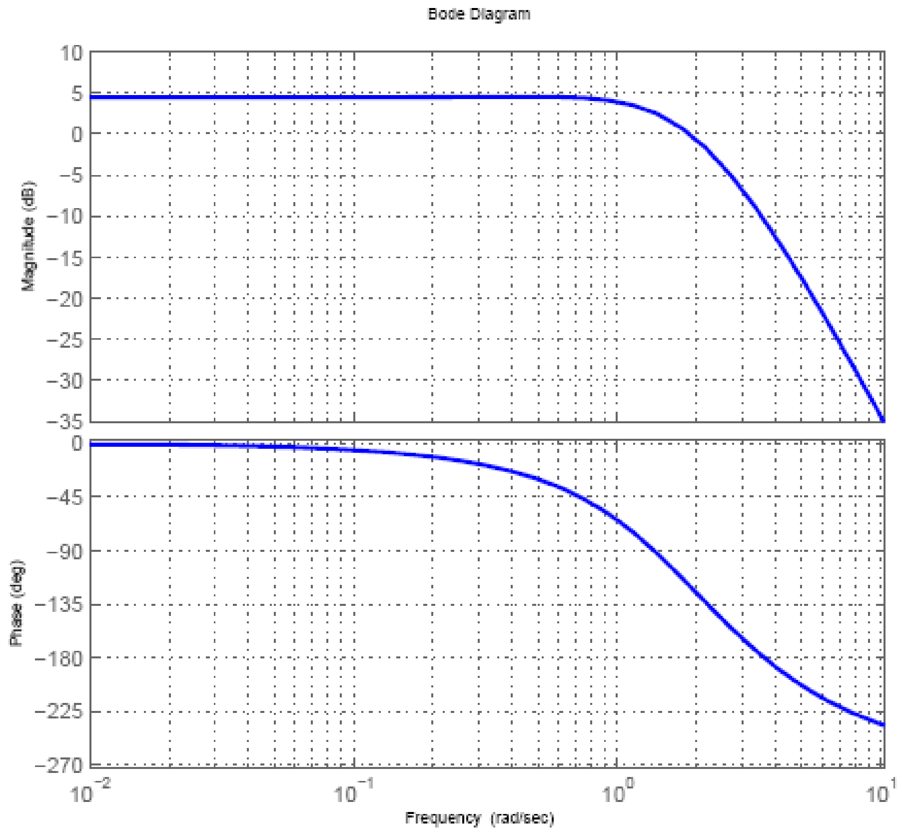
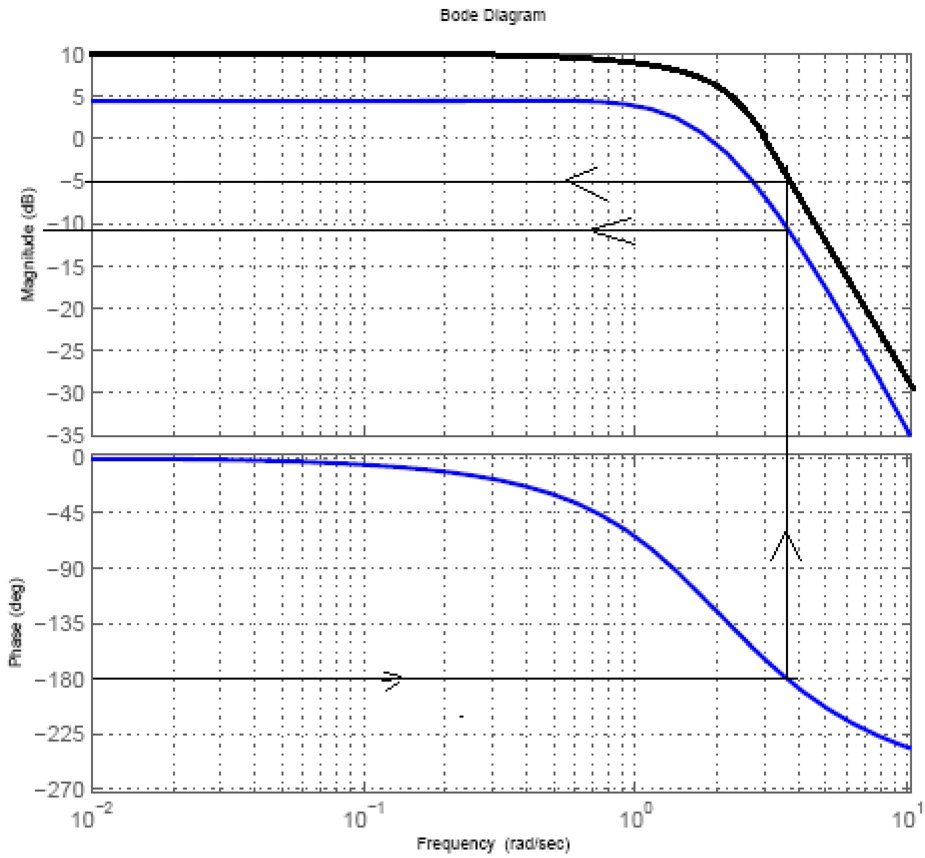


Figure 4

The proportional controller does not chsnge the phase but it does change then gain only

We can shift the Bode plot representing the gain 6db and keep a gain margin of at least 5db as shown in figure below



As ω tends to zero the gain approaches 10

$\therefore 10 = 20\log k_p \Rightarrow k_p = 3.16$

Before the controller $k_p = 10^{0.2} = 1.585$

The steady state error for unit input $e_{ss} = \dfrac{1}{1+k_p}$ as the system is zero type so

The steady state error is reduced from approximately 0.4 to approximately 0.25

4-b) Given the straight line Bode diagram of magnitude in figure 5, find the corresponding transfer function.

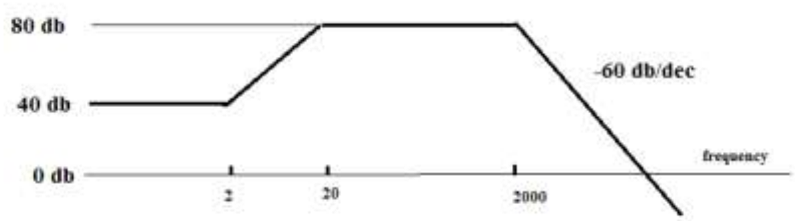


Figure 5

What is the type of the system?

Find the steady state error for unit input and ramp input.

Low frequency asymptotes $40db = 20\log x \Rightarrow x = 100$

At frequency $2s^{-1}$ the slope is 40db/dec, there is a factor $\left(\frac{s}{2}+1\right)^2$ in the numinator

At frequency $20s^{-1}$ the slope is 0db/dec, there is a factor $\left(\frac{s}{20}+1\right)^2$ in the denuminator

At frequency $2000s^{-1}$ the slope is -60db/dec, there is a factor $\left(\frac{s}{2000}+1\right)^3$ in the denuminator

$\therefore$ the transfer function G(s) is given by $G(s) = \dfrac{100(\frac{s}{2}+1)^2}{(\frac{s}{20}+1)^2(\frac{s}{2000}+1)^3}$

The system is type ZERO with K_p=100, K_v=0, K_a=0

The steady state error for unit input e_{ss}= $(1/(1+k_p))$ = 1/101

The steady state error for ramp input e_{ss}= $(1/k_v)$ =1/0 = ∞

GOOD LUCK