Benha University
Benha Faculty of Engineering
Electrical Engineering and Circuit Analysis(b) (E1102)
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2nd Term 2015/2016 تخلفات
Electrical Department
1st Year Electrical
Time: 1.5 Hrs

Model Answer

Question (1): **[15 Marks]**

Write a set of mesh-current equations that describe the circuit in Fig.1 in terms of the currents i_1 and i_2.

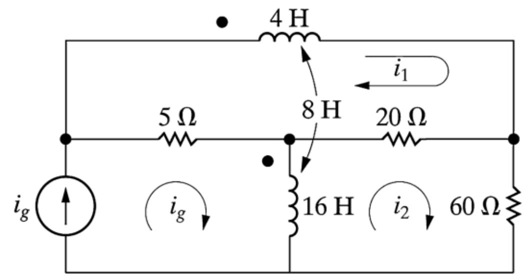


Fig.1

a) Summing the voltages around the i_1 mesh yie

$$4\frac{di_1}{dt} + 8\frac{d}{dt}(i_g - i_2) + 20(i_1 - i_2) + 5(i_1 - i_g)$$

The i_2 mesh equation is

$$20(i_2 - i_1) + 60i_2 + 16\frac{d}{dt}(i_2 - i_g) - 8\frac{di_1}{dt} =$$

Note that the voltage across the 4 H coil du
the current $(i_g - i_2)$, that is, $8d(i_g - i_2)/dt$,
voltage drop in the direction of i_1. The volt
induced in the 16 H coil by the current i_1, tha
$8di_1/dt$, is a voltage rise in the direction of i_2

<u>Question (2):</u> [15 Marks]

The switch in the circuit shown in Fig.2 has been in position ***a*** for a long time. At ***t = 0*** the switch is moved to position ***b***.

a) What is the initial value of v_c?

b) What is the final value of v_c?

c) What is the time constant of the circuit when the switch is in position ***b***?

d) What is the expression for $v_c(t)$ when $t \geq 0$?

e) What is the expression for $i(t)$ when $t \geq 0^+$?

f) How long after the switch is in position ***b*** does the capacitor voltage equal zero?

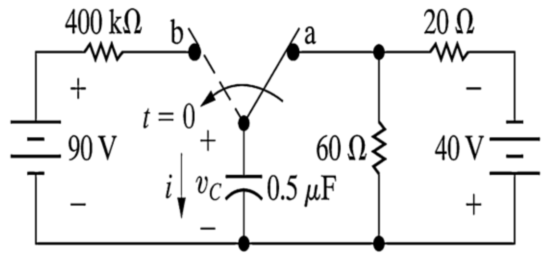


Fig.2

a) The switch has been in position a for a long time, so the capacitor looks like an open circuit. Therefore the voltage across the capacitor is the voltage across the 60 Ω resistor. From the voltage-divider rule, the voltage across the 60 Ω resistor is 40 × [60/(60 + 20)], or 30 V. As the reference for v_C is positive at the upper terminal of the capacitor, we have $v_C(0) = -30$ V.

b) After the switch has been in position b for a long time, the capacitor will look like an open circuit in terms of the 90 V source. Thus the final value of the capacitor voltage is + 90 V.

c) The time constant is

$$\tau = RC$$
$$= (400 \times 10^3)(0.5 \times 10^{-6})$$
$$= 0.2 \text{ s.}$$

d) Substituting the appropriate values for v_f, $v(0)$, and t into Eq. 7.60 yields

$$v_C(t) = 90 + (-30 - 90)e^{-5t}$$
$$= 90 - 120e^{-5t} \text{ V}, \quad t \geq 0.$$

e) Here the value for τ doesn't change. Thus we need to find only the initial and final values for the current in the capacitor. When obtaining the initial value, we must get the value of $i(0^+)$, because the current in the capacitor can change instantaneously. This current is equal to the current in the resistor, which from Ohm's law is $[90 - (-30)]/(400 \times 10^3) = 300\ \mu$A. Note that when applying Ohm's law we recognized that the capacitor voltage cannot change instantaneously. The final value of $i(t) = 0$, so

$$i(t) = 0 + (300 - 0)e^{-5t}$$
$$= 300e^{-5t}\ \mu\text{A}, \quad t \geq 0^+.$$

We could have obtained this solution by differentiating the solution in (d) and multiplying by the capacitance. You may want to do so for yourself. Note that this alternative approach to finding $i(t)$ also predicts the discontinuity at $t = 0$.

f) To find how long the switch must be in position b before the capacitor voltage becomes zero, we solve the equation derived in (d) for the time when $v_C(t) = 0$:

$$120e^{-5t} = 90 \quad \text{or} \quad e^{5t} = \frac{120}{90},$$

so

$$t = \frac{1}{5}\ln\left(\frac{4}{3}\right)$$
$$= 57.54 \text{ ms.}$$

Note that when $v_C = 0$, $i = 225\ \mu$A and the voltage drop across the 400 kΩ resistor is 90 V.

<u>Question (3):</u> [15 Marks]

In the circuit shown in Fig.3, $V_o = 0$, and $I_o = -12.25\ mA$.

a) Calculate the roots of the characteristic equation.

b) Calculate v and dv/dt at $t = 0^+$.

c) Calculate the voltage response for $t \geq 0$.

d) Plot $v(t)$ versus t for the time interval $0 \leq t \leq 11\ ms$.

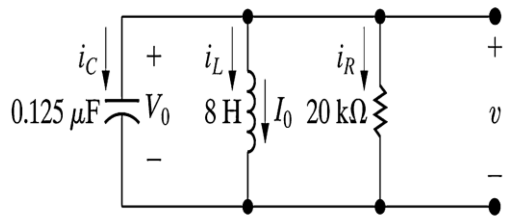


Fig.3

a) Because

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2(20)10^3(0.125)} = 200 \text{ rad/s},$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{10^6}{(8)(0.125)}} = 10^3 \text{ rad/s},$$

we have

$$\omega_0^2 > \alpha^2.$$

Therefore, the response is underdamped. Now,

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10^6 - 4 \times 10^4} = 100\sqrt{96}$$

$$= 979.80 \text{ rad/s},$$

$$s_1 = -\alpha + j\omega_d = -200 + j979.80 \text{ rad/s},$$

$$s_2 = -\alpha - j\omega_d = -200 - j979.80 \text{ rad/s}.$$

For the underdamped case, we do not ordinarily solve for s_1 and s_2 because we do not use them explicitly. However, this example emphasizes why s_1 and s_2 are known as complex frequencies.

b) Because v is the voltage across the terminals of a capacitor, we have

$$v(0) = v(0^+) = V_0 = 0.$$

Because $v(0^+) = 0$, the current in the resistive branch is zero at $t = 0^+$. Hence the current in the capacitor at $t = 0^+$ is the negative of the inductor current:

$$i_C(0^+) = -(-12.25) = 12.25 \text{ mA}.$$

Therefore the initial value of the derivative is

$$\frac{dv(0^+)}{dt} = \frac{(12.25)(10^{-3})}{(0.125)(10^{-6})} = 98{,}000 \text{ V/s}.$$

c) From Eqs. 8.30 and 8.31, $B_1 = 0$ and

$$B_2 = \frac{98{,}000}{\omega_d} \approx 100 \text{ V}.$$

Substituting the numerical values of α, ω_d, B_1, and B_2 into the expression for $v(t)$ gives

$$v(t) = 100e^{-200t} \sin 979.80t \text{ V}, \quad t \geq 0.$$

d) Figure 8.9 shows the plot of $v(t)$ versus t for the first 11 ms after the stored energy is released. It clearly indicates the damped oscillatory nature of the underdamped response. The voltage $v(t)$ approaches its final value, alternating between values that are greater than and less than the final value. Furthermore, these swings about the final value decrease exponentially with time.

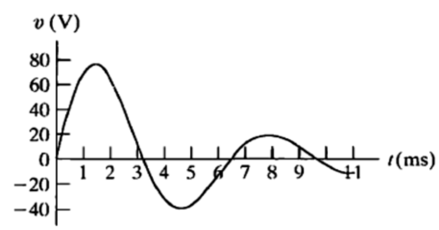


Figure 8.9 ▲ The voltage response for Example 8.4.

Question (4): [15 Marks]

Use the node voltage method to find the steady-state expression for $\boldsymbol{v_o}$ in the circuit seen in Fig.4 if v_g equals ***130 cos 10,000t V***.

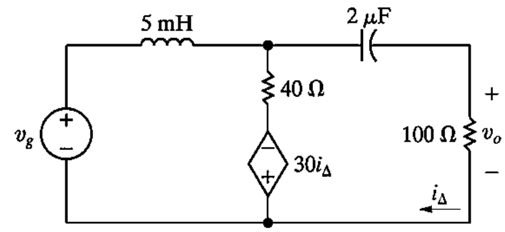


Fig.4

Solve by yourself.

Question (5): [15 Marks]

A factory has an electrical load of ***1600 kW*** at a lagging power factor of ***0.8***. An additional variable power factor load is to be added to the factory. The new load will add ***320 kW*** to the real power load of the factory. The power factor of the added load is to be adjusted so that the overall power factor of the factory is ***0.9*** lagging.

a) Specify the reactive power associated with the added load.

b) Does the added load absorb or deliver magnetizing ***vars***?

c) What is the power factor of the additional load?

d) Assume that the voltage at the input to the factory is ***2400 V (rms)***. What is the ***rms*** magnitude of the current into the factory before the variable power factor load is added?

e) What is the ***rms*** magnitude of the current into the factory after the variable power factor load has been added?

f) Comment on the answers of **d)** and **e)**.

[a] $S_o = \text{ original load } = 1600 + j\dfrac{1600}{0.8}(0.6) = 1600 + j1200\,\text{kVA}$

$$S_f = \text{ final load } = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560\,\text{kVA}$$

$$\therefore \quad Q_{\text{added}} = 560 - 1200 = -640\,\text{kVAR}$$

[b] deliver

[c] $S_a = \text{ added load } = 320 - j640 = 715.54\underline{/-63.43^\circ}\,\text{kVA}$

$$\text{pf } = \cos(-63.43) = 0.447 \text{ leading}$$

[d] $\mathbf{I}_L^* = \dfrac{(1600 + j1200) \times 10^3}{2400} = 666.67 + j500\,\text{A}$

$$\mathbf{I}_L = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_L| = 833.33\,\text{A(rms)}$$

[e] $\mathbf{I}_L^* = \dfrac{(1920 + j560) \times 10^3}{2400} = 800 + j233.33$

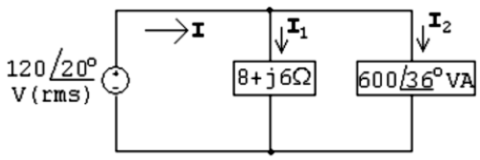
$$\mathbf{I}_L = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_L| = 833.33\,\text{A(rms)}$$

Question (6): **[15 Marks]**

In a balanced three-phase system, the source has an abc sequence, is Y-connected, and V_{an} = 120/20° V. The source feeds two loads, both of which are Y-connected. The impedance of load 1 is 8 + j6 Ω/φ. The complex power for the a-phase of load 2 is 600/36° VA. Find the total complex power supplied by the source.

The a-phase of the circuit is shown below:



$$\mathbf{I}_1 = \frac{120\angle 20^\circ}{8+j6} = 12\angle -16.87^\circ \text{A (rms)}$$

$$\mathbf{I}_2^* = \frac{600\angle 36^\circ}{120\angle 20^\circ} = 5\angle 16^\circ \text{A (rms)}$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 12\angle -16.87^\circ + 5\angle -16^\circ = 17\angle -16.61^\circ \text{ A (rms)}$$

$$S_a = \mathbf{VI}^* = (120\angle 20^\circ)(17\angle 16.61^\circ) = 2040\angle 36.61^\circ \text{ VA}$$

$$S_T = 3S_a = 6120\angle 36.61^\circ \text{ VA}$$

With best wishes