**Benha University**
**College of Engineering at Banha**
**Mechanical Eng. Dept.** **3rd Year Mechanics**
**Subject :Automatic Control (M 1352)** **May21/2016**
Questions For Corrective Examination

**Examiner : Dr. Mohamed Elsharnoby** **Time :180 min.**

**Attempt all questions, Number of questions = 4, Number of pages = 2**

1- a) For the block diagram shown in Figure 1, draw the equivalent signal flow graph using 5 nodes: input, output, and nodes a, b, and c.
Use masson rule to find the transfer function of the system.

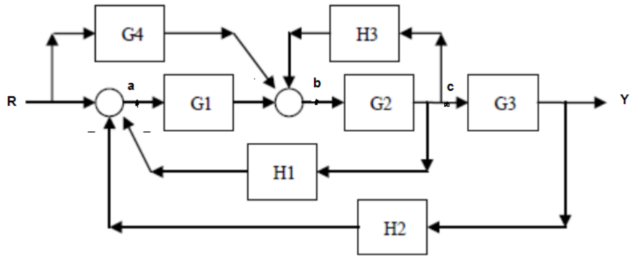



Figure 1

1-b)
If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{S^2 + 2S + 1}{S^4 + 2S^3 + 5S^2 + 3S + 7}$$

i) Draw a signal flow graph represents this system.
ii) Deduce the state space representation of the system.
iii) Write the differential equation of the system in canonical form.

2-a) Consider the system below in figure 2 and **let p=0**.

i) Determine conditions on K and z so that the system is stable.
ii) Determine all possible conditions on K and z so that the system will be marginally stable

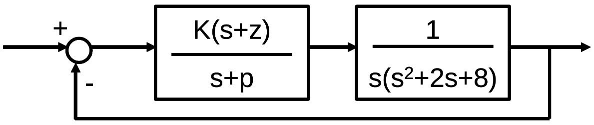



Figure 2

2-b Determine whether the standard feedback system is stable for the following cases, and justify your answers. If the feedback system is unstable determine how many of its poles are in the right half plane.

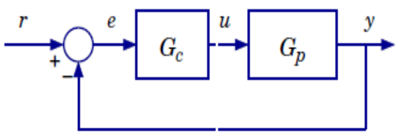



Figure 3

(a) $G_c(s) = \frac{10}{(s+1)}$; $G_p(s) = \frac{2}{(s+1)(s+2)}$

(b) $G_c(s) = \frac{5\ (s+1)}{s}$; $G_p(s) = \frac{1}{s-2}$

(c) $G_c(s) = \frac{5\ (0.1s+1)}{s}$; $G_p(s) = \frac{(s-2)^2}{(s^2+6s+10)(s+5)}$

b) What is the type of the system for the given three casees, determine the steady state error for unit step, ramp, and acceleration inputs.

3-a) Consider the standard feedback system shown above in Figure 3. Let the plant be given as

$$G_p(s) = \frac{1}{s(s+5)}.$$

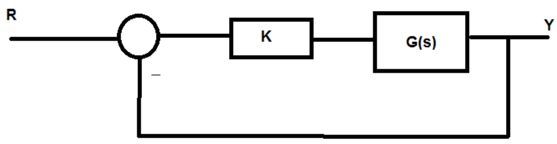
Design a Proportional plus Integral (PI) controller of the form

$$G_c(s) = K_P + \frac{K_I}{s}$$

such that the closed loop system is stable with one pole at $r_1 = -4$ and the other two poles are at $r_{2,3} = -\zeta\omega_o \pm j\omega_o\sqrt{1-\zeta^2}$ with $\omega_o \geq 2$, $\zeta > 0$, and $\zeta$ is as large as possible.

ii) Find $t_r$ , $t_p$ , $t_s$, $t_d$ , and $M_p$ for $\zeta = 0.707$ and $\omega_o = 2.5$ rad/sec

3-b) For the system shown in figure 4 with K > 0 and $G(s) = \frac{2-3s}{s^2+s+1}$.




i) Sketch the Nyquist plot for G(iω) . Calculate the real axis crossings and the corresponding frequencies.

ii) By using the Nyquist plot for G(iω) , find the range of k such that the closed loop system is stable.

Figure 4

4-a) Sketch the root locus for the system given in figure 3 with

$$G_c(s) = K\frac{(s+1)}{s} \qquad G_p(s) = \frac{1}{s(s+7)^2}$$

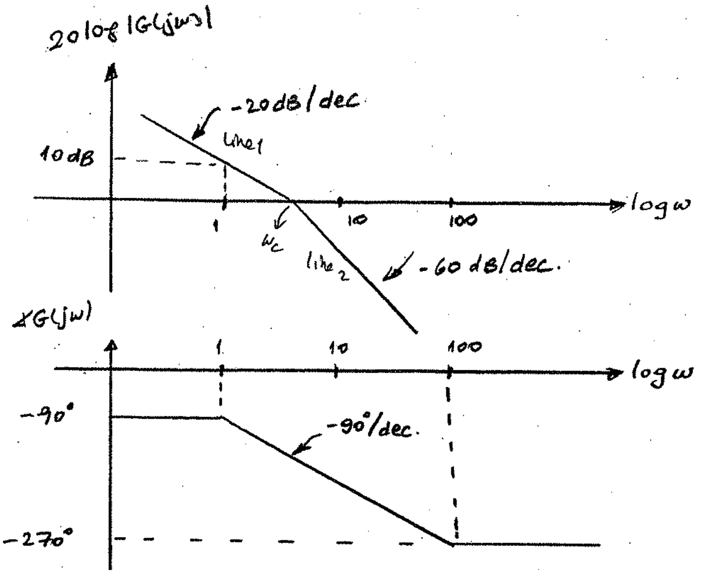
Make sure you provide verbal description on the following: open-loop pole/zero map; real axis decision; and asymptotes. Find the jw-axis crossing point and the corresponding value of K.

4-b) Consider the following system where G(s) is a transfer function. Asymptotic Bode plots of G(s) is given in figure 5 below. For calculations, you may use these Asymptotic plots.

i) Find the gain margin of the system.

ii) Find the phase margin of the system.

iii) Find the transfer function of the system. .



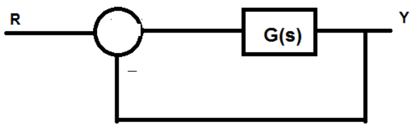




Figure 5

Note : justfy your answer

G O O D L U C K

**Benha University**
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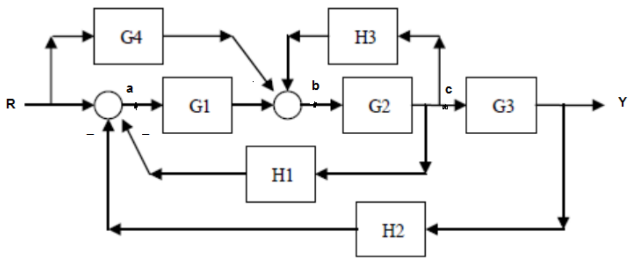
**Mechanical Eng. Dept.** **4th Year Mechanics**
**Subject :Automatic Control** **May 21**/2016
Model Answer of the Final Examination
**Elaborated by: Dr. Mohamed Elsharnoby**

**نموذج اجابة**

**المادة : التحكم الآلى م 1352**

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

1-a)

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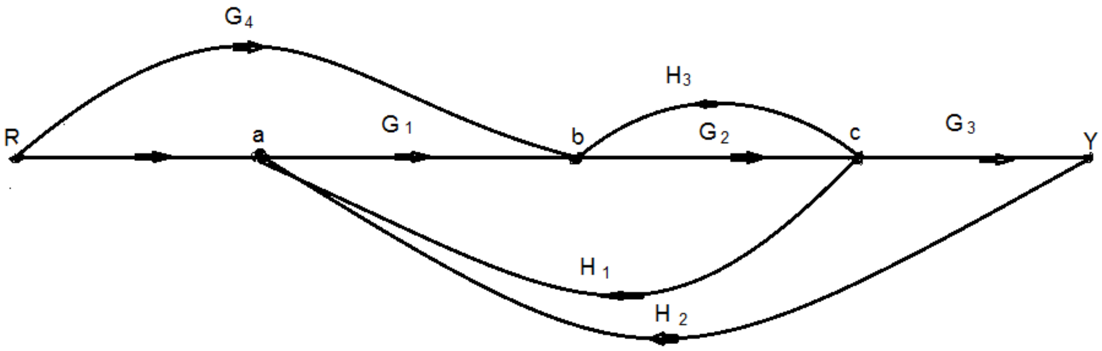



L
Loops
$L_1 = -G_1G_2H_1$ $L_2 = -G_1G_2G_3H_2$ $L_3 = -G_2H_3$
Paths
$M_1 = G_1G_2G_3$ $M_2 = G_4G_2G_3$
$\Delta = 1 + G_1G_2H_1 + G_1G_2G_3H_2 + G_2H_3$
$\Delta_1 = 1$
$\Delta_2 = 1$
$TF = (M_1\,\Delta_1 + M_2\,\Delta_2)/\,\Delta = (G_1G_2G_3 + G_4G_2G_3)/(1 + G_1G_2H_1 + G_1G_2G_3H_2 + G_2H_3)$

2-a)
FOR P=0, the characteristic equation is given by:
$S^2(S^2+2S+8) + K(S+Z) = 0$
$S^4+2S^3+8S^2+KS+KZ=0$
Construct the Huwarth array

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 8 | KZ |
| $S^3$ | 2 | K | 0 |
| $S^2$ | (16-k)/2 | KZ | 0 |
| S | $\frac{(16k-k^2-2kz)(2)}{16-k}$ | 0 | |
| $S^0$ | kz | | |

For stable systen

$16 > K > 0$, $Z>0$

$\frac{(16k-k^2-2kz)(2)}{16-k} > 0$

$16-K-2z > 0$

$16 > k + 2z$

ii For marginally stable system

Put $S = j\omega$

$\omega^4 - 2j\omega^3 - 8\omega^2 + jk\omega + kz = 0$

$-2\omega^3 + k\omega = 0$ $\quad k = 2\omega^2$

$\omega^4 - 8\omega^2 + kz = 0$

$\omega^4 + (2z-8)\omega^2 + = 0$

$\omega^4 + (2z-8)\omega^2 + = 0$ $\quad \omega = 0.$ $\quad \omega = \pm (8-2z)^{1/2}$ , $K = 16-4z$

$4 > z > 0$ , $16 > k > 0$

2-b

In all cases we need to apply the Routh-Hurwitz test

(a) $(s+1)^2(s+2)+20 = (s^2+2s+1)(s+2)+20 = s^3+4s^2+5s+22$

| | | |
|---|---|---|
| $s^3$ | 1 | 5 |
| $s^2$ | 4 | 22 |
| $s^1$ | -2 | 0 |
| $s^0$ | 22 | |

two sign changes in the first column of the Routh table ⇒ feedback system is unstable with two poles in the right half plane

b) $s(s-2)+5(s+1) = s^2+3s+5$ → all coefficients are positive in a second order polynomial ⇒ feedback system is stable.

c) $s(s^2+6s+10)(s+5)+0.5(s+10)(s-2)^2 = s^4+11s^3+40s^2+50s+0.5(s+10)(s^2-4s+4)$

$= s^4+10.5s^3+37s^2+32s+20$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 37 | 20 |
| $s^3$ | 10.5 | 32 | 0 |
| $s^2$ | $\left(37-\frac{32}{10.5}\right)$ | 20 | 0 |
| $s^1$ | $x$ | 0 | |
| $s^0$ | 20 | | |

$x = \left(\left(37-\frac{32}{10.5}\right)\times 32 - 20\times 10.5\right) \Big/ \left(37-\frac{32}{10.5}\right)$

$x \geq \frac{33\times 32 - 20\times 10.5}{\left(37-\frac{32}{10.5}\right)} > 0$

feedback system is stable.

2-b) System (a) is type zero with $k_P = 10$ . $k_v = ,0$ $\quad k_a = 0,$

And the steady state errors for unit step, ramp, and acceleration inputs are respectively 1/11, ∞, ∞.

System (b) is type one with $k_P = \infty$ . $k_v = ,-2.5$ $\quad k_a = 0,$

And the steady state errors for unit step, ramp, and acceleration inputs are respectively 0, -0.4, ∞.

System (c) is type one with $k_P = \infty$ . $k_v = ,0.4$, $k_a = 0,$

And the steady state errors for unit step, ramp, and acceleration inputs are respectively 0, 2.5, ∞.

3-a)

the open loop transfer function $G_c G_p = \dfrac{K_p S + K_I}{S^2 (S+5)}$

THe closed loop transfer function = $\dfrac{K_p S + K_I}{S^3 + 5S^2 + K_p S + K_I}$

S = -4 satisfies the characteristic equation, leads to
$-64 + 80 - 4K_p + K_I = 0$, $K_I = 4K_p - 16$
The characteristic equation can be written in the following form:
$(S+4)(S^2 + 2\zeta\omega_o S + \omega_o^2)$ , where $\omega_o^2 = K_I/4$, $2\zeta\omega_o = 1$,
For $\omega_o \geq 2$, $K_I \geq 16$, $K_p \geq 8$, $\zeta \leq 0.25$
Values $\omega_o = 2$, $K_I = 16$, $K_p = 8$, $\zeta = 0.25$

ii) $t_r \Longrightarrow t_r = \dfrac{\pi - \theta}{\omega_d}, \theta = \cos^{-1}\zeta =$ ,

$t_p \Longrightarrow t_p = \pi / \omega_d = \pi / \omega_n \sqrt{1-\zeta^2} =$ ,

$t_s = \Rightarrow t_s \cong \dfrac{4}{\sigma} = 4T$ = 2.3 sec,

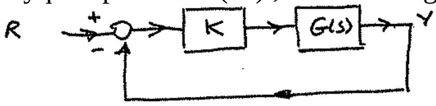
$t_d =$ ,

$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% =$

3-b)

For the system shown in figure 4 with K > 0 and $G(s) = \frac{2-3s}{s^2+s+1}$.

i) Sketch the Nyquist plot for G(iω) . Calculate the real axis crossings and the corresponding frequencies.
ii) By using the Nyquist plot for G(iω) , find the range of k such that the closed loop system is stable.



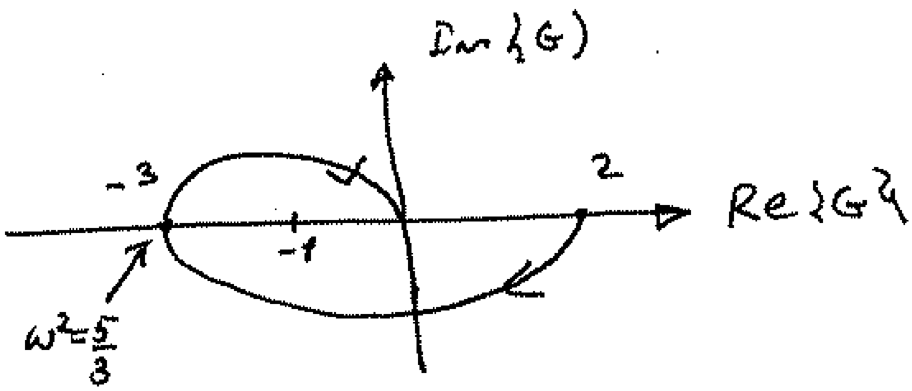

(02)

i) $G(j\omega) = \dfrac{2-3j\omega}{1-\omega^2+j\omega} = \dfrac{(2-5\omega^2) - j\omega(5-3\omega^2)}{(1-\omega^2)^2+\omega^2}$ (02)

$\omega = 0 \Rightarrow Re\{G\} = 2$ $Im\{G\} = 0$

$\omega^2 = \frac{5}{3} \Rightarrow Re\{G\} = -3$ $Im\{G\} = 0$ (03)

For $0 < \omega < \sqrt{\frac{5}{3}} \Rightarrow Im\{G\} < 0$ , $\omega > \sqrt{\frac{5}{3}} \rightarrow Im\{G\} > 0$ and $G(j\omega) \rightarrow 0$ as $\omega \rightarrow \infty$




(03)

ii) For stability, real axis crossing should be to the ~~left~~ right of -1 i.e.

$-3K > -1 \Rightarrow$ $\boxed{0 < K < 1/3}$ (05/05)

4-a)

i) Open loop poles are at 0 ,0 , -7, -7.
ii) Open loop zero at -1.
iii) Number of asymptotes = 4 – 1 = 3.
iv) Angles of asymptotes with real axis are $60^{o}$ ,$180^{o}$, $300^{o}$.
v) Intersection point of asymptotes on the real line = (-7-7+1)3= -13/3
vi) Intersection with the imaginary axis

The characteristic equation is given by :

$S^2(S+7)^2 + K (S+1) = 0$

$S^4+4S^3+49S^2+kS+K=0$

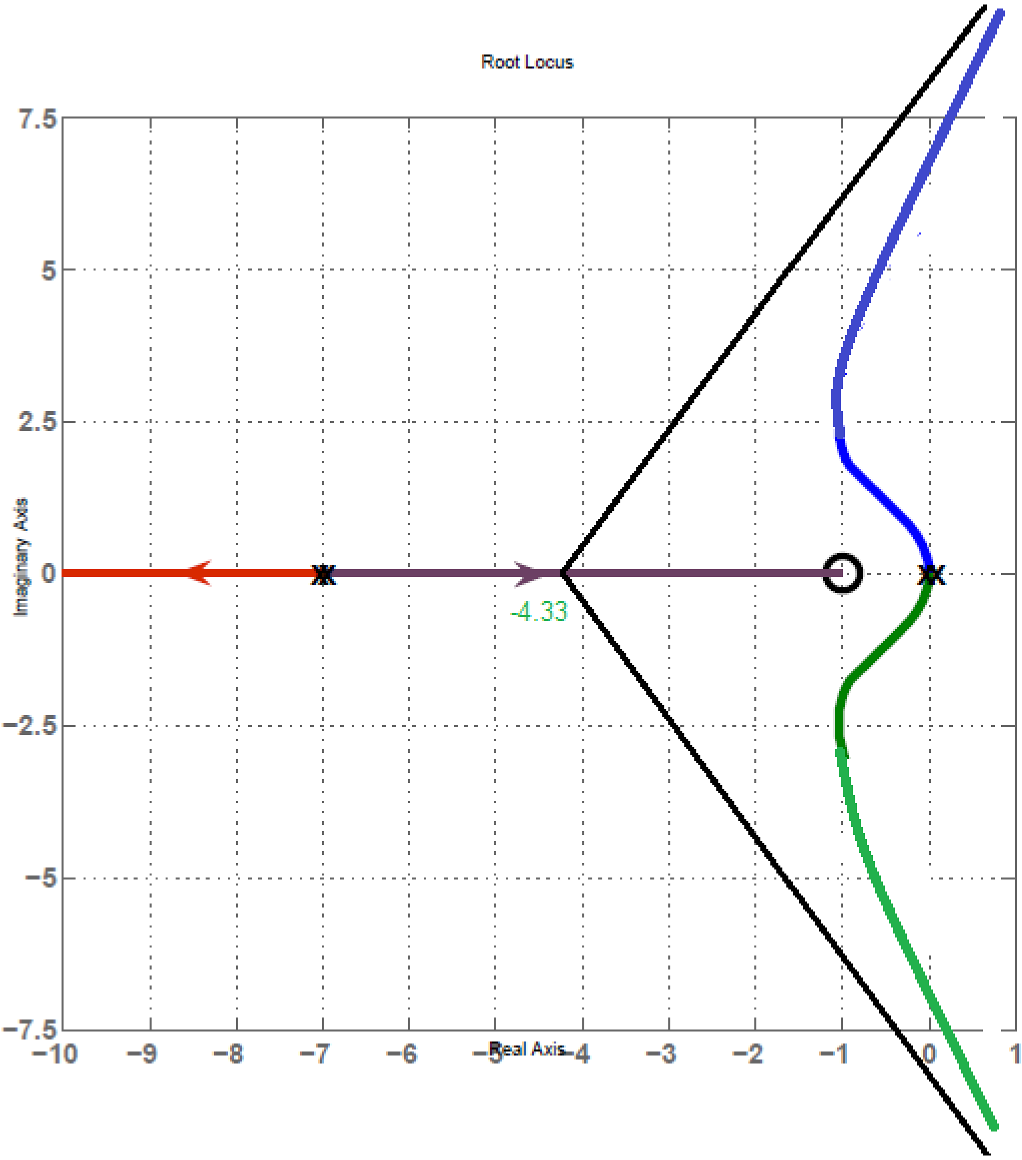
Put $S = j\omega$

$\omega^4 -4j\omega^3-49\omega^2+jk\omega +k=0$

$-4 \omega^3+k\omega = 0$ $\qquad$ $k=4 \omega^2$

$\omega^4 -49\omega^2+k=0$

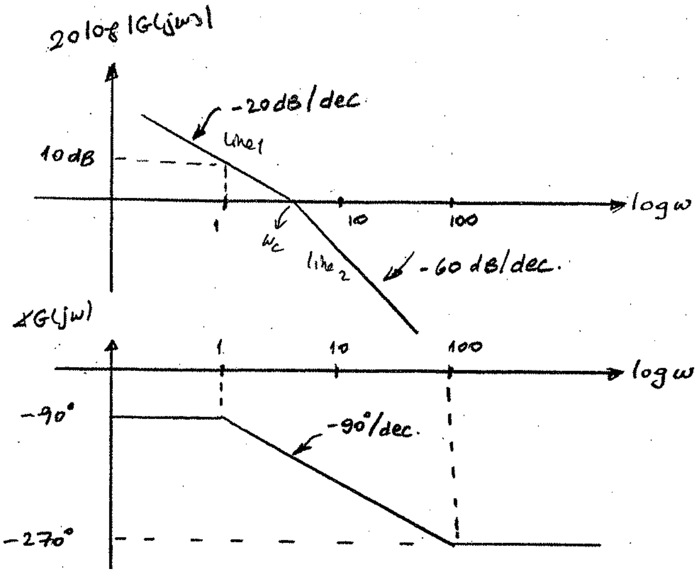
$\omega^4 -45\omega^2=0 \quad \omega=0 \quad \omega=\pm (45)^{1/2} \quad K = 180$

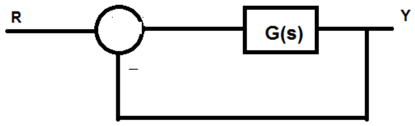



$\omega^4$
-4
+k

4-b) Consider the following system where G(s) is a transfer function. Asymptotic Bode plots of G(s) is given in figure 5 below. For calculations, you may use these Asymptotic plots.

iii) Find the gain margin of the system.
iv) Find the phase margin of the system.
v) Find the transfer function of the system. .




line 1 ⇒ $20\log K - 20\log \omega$ ⇒ $\omega = 1 \Rightarrow 10\,dB \Rightarrow 20\log K = 10$ ⇒ $\boxed{K = \sqrt{10}}$

⇒ $20\log K - 20\log \omega_c = 0$ ⇒ $\boxed{\omega_c = K = \sqrt{10}}$

line 2 ⇒ $A - 60\log \omega = 0$ at $\omega_c = \sqrt{10}$ ⇒ $A - 60\log\sqrt{10} = 0$ ⇒ $A = 30$ dB. (05)

i) at $\omega_c = \sqrt{10}$ ⇒ $\angle = -90 - 90\log\omega_c = -90 - 45^\circ = -135^\circ$ ⇒ $\boxed{PM = 180 - 135 = 45^\circ}$

ii) $-90 - 90\log\omega_o = -180$ ⇒ $\omega_o = 10$ rad/sec. ⇒ $30 - 60\log 10 = -30\,dB$

⇒ $\boxed{GM = 30\,dB}$ (05)

iii) $$G(s) = \frac{K}{s\left(\frac{s}{\omega_c}+1\right)^2} = \frac{\sqrt{10}}{s\left(\frac{s}{\sqrt{10}}+1\right)^2} = \frac{3.16}{s\,(0.32s+1)^2}$$ (05)