جامعة بنها - كلية الهندسة ببنها - قسم الهندسة الكهربية
نموذج الإجابة امتحان مادة هندسة التحكم ك352 تخلفات مايو السبت 22-5-2016
مدرس بالقسم شوقي حامد عرفه

Benha University,	Elect.Eng.Depart.
Faculty of Engineering	Third Year 22-5-2016
Subject: Control Engineering (E352)	Time: 3-hours تخلفات

Q1 (10 mark)

a- Define: gain margin- phase margin?

b- Define: $\boldsymbol{\omega_n}$, $\boldsymbol{\omega_d}$, $\boldsymbol{\omega_r}$, $\boldsymbol{\omega_c}$, $\boldsymbol{\omega_g}$, $\boldsymbol{\omega_p}$, $\mathbf{M_r}$, $\boldsymbol{\eta}$?

c- Consider a control system shown in Fig.1 if **G(S) =K/[(S^2+8S+7)], H(S) =1/(S+2).** Find the **open loop** transfer function and the **closed loop** transfer function?

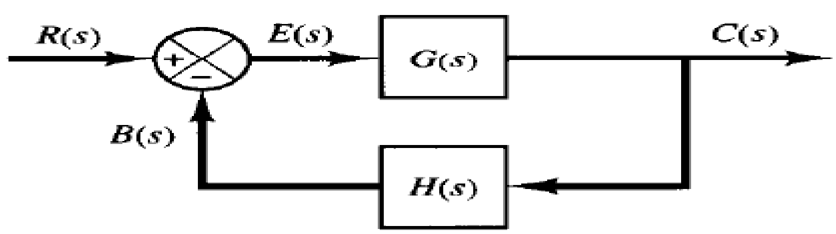


Fig.1

Q2 (15 marks)

Consider a unity feedback control system shown in Fig.1 if $G(s) = \frac{16}{S(S+4)}$

a-Find M_r and ω_r ?

b- Find the frequency response as r(t)=3sinωt ?

Q3 (15 marks)

Consider a control system shown in Fig.1 if

$$\mathrm{G(s)H(s)} = \frac{\mathrm{k}}{(\mathrm{S}+1+\mathrm{j})(\mathrm{S}+2)(\mathrm{S}+1-\mathrm{j})} = \frac{\mathrm{k}}{\mathrm{S}^3+4\mathrm{S}^2+6\mathrm{S}+4}$$

a- Sketch the **complete root locus** for positive values of **K**?

b- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles** using **the plot**?

c- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles analytically**?

d- Write short MATLAB program to solve a & b?

Q4 (20 marks)

Consider a control system shown in Fig.1 if

$$G(s)H(s) = \frac{10}{(S+1+j)(S+2)(S+1-j)} = \frac{10}{S^3+4S^2+6S+4}$$

a- Prove that the gain margin=**6.02 db at 2.45 rad/sec**. and the phase margin= **30.3 degrees at 1.78 rad/sec**.?

b- Sketch the **polar plot?**

c- Sketch the **Bode plot?**

d- Show the gain margin and the phase margin on **the plots**?

e- Write short MATLAB program to solve a , b and C?

Answer

Q1 (10 marks)

a-Define: gain margin- phase margin?

-Gain margin G_m: it is reciprocal of the magnitude of the output frequency response at the Phase crossover frequency ω_p

a- G_m=1/[Real of G(j ω_p)H(j ω_p)]= 1/|G(j ωp)H(j ωp)| =K_c/K

G_M=20log G_m db

Phase margin γ_m: it is the angle of the output frequency response at the -b gain crossover frequency plus 180 degrees -c

$$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180 \text{ deg.}$$

b- Define: **ω_n , ω_d , ω_r , ω_c , ω_g , ω_p , M_r, η**? -d

-Natural frequency ω_n rad/sec**:** it is the natural frequency depends on the natural of the system parameters.

- Under damped natural frequency ω_d rad/sec**:** it is the under damped natural frequency depends on the damping coefficient **η** as it is less than one **η <**1.

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

-Resonant frequency ω_r rad/sec**:** it is the frequency at which the peak value of the output frequency response for a second order is occurred and it is equal to

$$\boldsymbol{\omega_r = \omega_n\sqrt{1-2\zeta^2}\ , 0 \le \zeta \le 0.707}$$

As ζ approaches zero, M_r approaches infinity

$0 < \zeta \le 0.707$, the resonant frequency ω_r is less than the damped natural frequency

-Corner frequency ω_c rad/sec**:** it is the frequency at which the magnitude of the output frequency response is changed sharply. It may be (0, 1, 1/T, $\boldsymbol{\omega_n}$**)**

-Gain crossover frequency ω_g: it is the frequency at which the magnitude of the output frequency response is equal to one or zero decibel.

$|G(j\,\boldsymbol{\omega g})H(j\,\boldsymbol{\omega g})\,| = 1$ or $|G(j\,\boldsymbol{\omega g})H(j\,\boldsymbol{\omega g})\,| = 0db$

-Phase crossover frequency ω_p: it is the frequency at which the phase of the output frequency response is equal to (-180) degrees.

Imag. [G(j ω_p)H(j ω_p)]=0 or $\angle G(j\,\boldsymbol{\omega p})H(j\,\boldsymbol{\omega p}) = -180\text{deg}.$

-Maximum resonant magnitude M_r: it is the peak value of the output frequency response for a second order system $\boldsymbol{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}}$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad M_r = |G(j\omega)|_{max} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

-damping coefficient η it depends on the natural of the system parameters. For second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Values of η	**System stability**	**Step-response**
0> η	System is unstable	undefined
η =0	System is critically stable	oscillatory
0< η <1	System is stable	Under-damped
0< η =1	System is stable	Critically damped

0<η >1	System is stable	Over damped

c- Consider a control system shown in Fig.1 if **G(S) =K/[(S^2+8S+7)], H(S) =1/(S+2).** Find the **open loop** transfer function and the **closed loop** transfer function?

The open loop TF=G(s) H(s)= **K/[(S^2+8S+7)(S+2)]**

$$\text{Closed loop tf} = \frac{C(S)}{R(S)} = \frac{\mathrm{G(s)}}{1+G(S)H(S)} = \frac{\mathrm{K(S+2)}}{\mathrm{S^3+10S^2+23S+14+K}}$$

Q2 (15 marks)

Consider a unity feedback control system shown in Fig.1 if $G(s) = \frac{16}{S(S+4)}$

a-Find M_r *and* ω_r ?

$$\frac{C(S)}{R(S)} = \frac{\mathrm{G(s)}}{1+G(S)H(S)} = \frac{{\omega_n}^2}{S^2+2\eta\omega_n S+{\omega_n}^2} = \frac{16}{S^2+4S+16}$$

$$\omega_n = 4\,\mathrm{rad/sec.},\ \ \eta = 0.5, M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2(0.5)\sqrt{1-(0.5)^2}} = 1.155$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} = 4\sqrt{1-2(0.5)^2} = 2.818\,\mathrm{rad/sec.}$$

b- Find the frequency response as r(t)=3sinωt ?

Steps to find frequency Response:

1- the closed loop transfer function =**T(s)=C(S)/R(S)** =

$$\frac{C(S)}{R(S)} = \frac{\mathrm{G(s)}}{1+G(S)H(S)} = \frac{{\omega_n}^2}{S^2+2\eta\omega_n S+{\omega_n}^2} = \frac{16}{S^2+4S+16}$$

2-the closed loop frequency transfer function =

$$\mathbf{T\,(j\omega)=C(j\omega)/R(j\omega)} = \frac{16}{(\mathrm{j}\omega)^2+4(\mathrm{j}\omega)+16} = \mathrm{M}\llcorner\Phi=\mathrm{Re+j\ imag}$$

$$\boldsymbol{M} = \frac{\mathbf{16}}{\sqrt{(\mathbf{16}-\omega^2)^2+\mathbf{16}\omega^2}} \quad , \ \boldsymbol{\Phi} = \mathbf{tan}^{-1}[\mathbf{4}\boldsymbol{\omega}\,/(\mathbf{16}-\boldsymbol{\omega}^2)]$$

3-As the input =$r(t) = 3sin\omega t$ then the response = $C(t) = 3M\sin(\omega t + \Phi)$

$$= \frac{\mathbf{48}}{\sqrt{(\mathbf{16}-\boldsymbol{\omega}^2)^2 + \mathbf{16}\boldsymbol{\omega}^2}} \sin[\omega t + \mathbf{tan}^{-1}[\mathbf{4}\boldsymbol{\omega}/(\mathbf{16}-\boldsymbol{\omega}^2)]$$

Q3 (15 marks)

Consider a control system shown in Fig.1 if

$$G(s)H(s) = \frac{k}{(S+1-j)(S+2)(S+1+j)} = \frac{k}{S^3+4S^2+6S+4}$$

a- Sketch the **complete root locus** for positive values of **K**?
b- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles** using **the plot**?
c- Find **K** that makes the complex closed loop poles have a damping ratio =**0.5** and **find the closed loop poles analytically**?
d- Write short MATLAB program to solve a & b?

a-Root locus:

1-the root locus is symmetrical about the real axis in the S-plane

2-the open loop $G(s)H(s) = \frac{k}{(S+1-j)(S+2)(S+1+j)} = \frac{k}{S^3+4S^2+6S+4}$

3-the root locus starts at the pole and ends at the zeros or infinity

4-number of root loci= n=number of poles of the open loop TF =3 at [-1+j,-1-j,-2]

5-number of zeros= m=0

6-number of asymptotes = n-m=3-0=3

8-center of gravity = $A = \frac{\sum poles - \sum zoles}{n-m} = \frac{-1-1-2}{3} = -1.3$point of intersection of asymptotes with real axis=

9-angles of asymptotes are = $\Theta = \frac{\pm 180(2R+1)}{n-m} = \pm 60$,±180

10- Points of crossing the imaginary axis as Routh test

Charct.equa=1+G(S)H(S)=0= $S^3+4S^2+6s+4+K$

S^3	1	6	4+K≥0, [20-K]/ 4≥0 then -4≤K≤20,Kc=20 $4S^2+24=0$, $S=\pm j\,\omega=\pm\sqrt{6}$ rad/sec
S^2	4	4+K	
S	[24-4-K]/ 4		
S^0	4+K		

11- there is no break points (break away or break in) at

$$-\frac{dK}{dS} = 0 = \frac{d}{dS}\left[\frac{1}{G(S)H(S)}\right] = \frac{d}{dS}\left[S^3 + 4S^2 + 6s + 4\right] = 3S^2 + 8s + 6 = 0$$

12- There is no break angles [±180(2R+1)/r] where r=number of branches(poles for break away or zeros for break in) R=0,1,-----no break angles

13-the angle of departures (complex poles) =

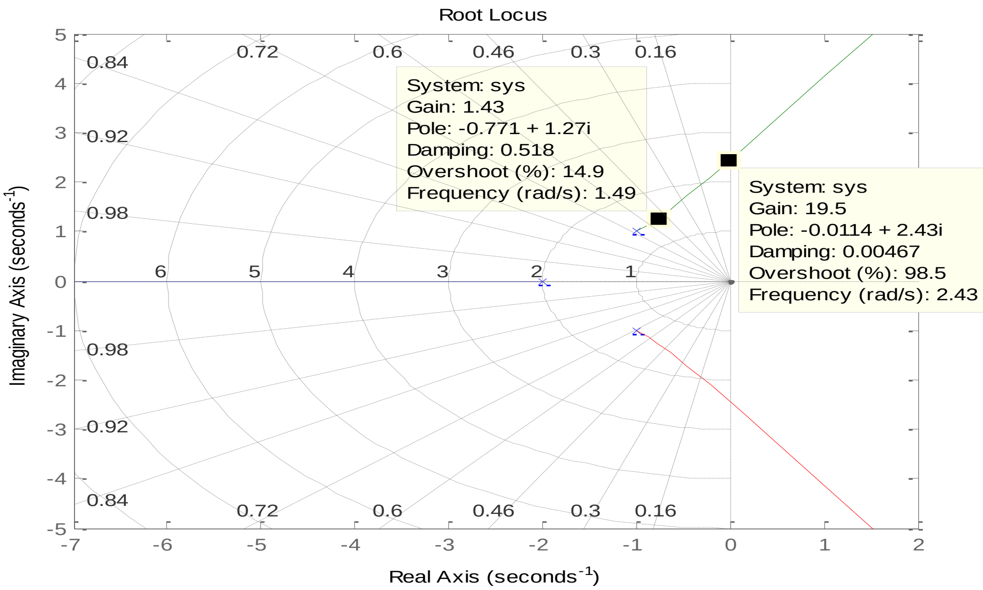
Angle of departure from a complex pole = 180°
− (sum of the angles of vectors to a complex pole in question from other poles)
+ (sum of the angles of vectors to a complex pole in question from zeros)

angle of departure=±180-90-45=±45deg

14-angle of arrival (complex zeros) as

Angle of arrival at a complex zero = 180°
− (sum of the angles of vectors to a complex zero in question from other zeros)
+ (sum of the angles of vectors to a complex zero in question from poles)

15-sketch the root loci as



16- the damping factor or coefficient ζ is straight line with slope $\Theta = \cos^{-1}\zeta$

with respect to the negative real axis in the S-plane. $\Theta = \cos^{-1} 0.5 = 60$deg. at the test point (intersection point) $S_d = -0.8 \pm j1.3$

$$\text{angle condition} = \sum_{n=1}^{n=3} [\Theta_{zeros} - \Theta_{poles}] = \pm 180(2R+1) = 90 + 54 + 36 = 180 \text{ deg}$$

$$\text{magnitude condition} = \sum_{n=1}^{n=3} \frac{\|\text{poles}\|}{\|\text{zeros}\|} = K = \ * \ * \ = 1.5$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = \sum_{n=1}^{n=3} closed\ loop\ poles = \text{constant as } n - m \geq 2$$

$$\sum_{n=1}^{n=3} open\ loop\ poles = -1 - 2 - 1 = -4 = \sum_{n=1}^{n=3} closed\ loop\ poles$$
$$= (-0.8 + j1.3, -0.8 - j1.3, p)$$

$$\text{then } p = -2.4 \text{ i.e. closed loop poles are } [-0.8 \pm j1.3, -2.4\,]$$

19- To find analytically closed loop poles and K as

$(S^2 + 2\zeta\omega_n S + \omega_n^2)(S+a)$=characteristic equa. for a third order syst.

Solve $1+G(S)H(S)=0= S^3+4S^2+6s+4+K=(S^2+ \omega_n S+ \omega_n^2)(S+a)$

$= S^3+(\omega_n +a)S^2+(\omega_n a+ \omega_n^2)S+ \omega_n^2 a$

$\omega_n +a =4$, $\omega_n a+ \omega_n^2=6$, $\omega_n^2 a=k+4$,then $\omega_n=1.5$rad/se., a=2.5,k=1.74

Prog. >>n=[1];d=[1 4 6 4]; rlocus(n,d), grid

Q4 (20 marks)

Consider a control system shown in Fig.1 if

$$G(s)H(s) = \frac{10}{(S+1+j)(S+2)(S+1-j)} = \frac{10}{S^3+4S^2+6S+4}$$

a- Prove that the gain margin=**6.02 db at 2.45 rad/sec**. and the phase margin= **30.3 degrees at 1.78 rad/sec**.?
b- Sketch the **polar plot?**
c- Sketch the **Bode plot?**
d- Show the gain margin and the phase margin on **the plots**?
e- Write short MATLAB program to solve a , b and C?

1- the open loop TF=G(s) H(s)= **G(S) H(S)**

$$G(s)H(s) = \frac{10}{(S+1+j)(S+2)(S+1-j)} = \frac{10}{S^3+4S^2+6S+4}$$

2- Find the freq.open loop TF=

G(jω)H(jω)$=\frac{10}{S^3+4S^2+6S+4} = Me^{j\Phi} = M\angle\Phi$ =Re+j imag

$$\boldsymbol{M} = \frac{\mathbf{10}}{\sqrt{(\mathbf{4-4\omega^2})^2+(\mathbf{6\omega-\omega^3})^2}} \quad , \boldsymbol{\Phi} = -\mathbf{tan^{-1}}((\mathbf{6\omega - \omega^3})/(\mathbf{4-4\omega^2}))$$

$$M = \frac{\mathbf{10}}{\sqrt{(\mathbf{4-4\omega^2})^2+(\mathbf{6\omega-\omega^3})^2}} = \frac{\mathbf{10}}{\sqrt{(\mathbf{4-4\omega^2})^2+(\mathbf{6\omega-\omega^3})^2}} = 1$$

$M = \frac{10}{\sqrt{(4-4(1.78)^2)^2+(6(1.78)-(1.78)^3)^2}} = 1,$ then $\omega_g = 1.78 rad/sec.$

$\Phi = -\tan^{-1}((6\omega - \omega^3)/(4-4\omega^2)) = -\tan^{-1}((6*2.45-2.45^3)/(4-4*2.45^2)) =$ -180 deg.

then $\omega_p = 2.45 rad/sec.$

$$M = \frac{10}{\sqrt{(4-4(2.45)^2)^2 + (6(2.45)-(2.45)^3)^2}} = 0.5, \text{then } G_M = 20log\frac{1}{0.5} = 6.02db$$

$$\Phi = -\tan^{-1}((6\omega - \omega^3)/(4-4\omega^2)) = -\tan^{-1}((6*1.78-1.78^3)/(4-4*1.78^2)) = -149.7\text{deg}. \quad .$$

$$\gamma_m = \angle G(j\,\omega_g)H(j\,\omega_g) + 180\text{ deg.} = 180 - 149.7 = 30.3deg.$$

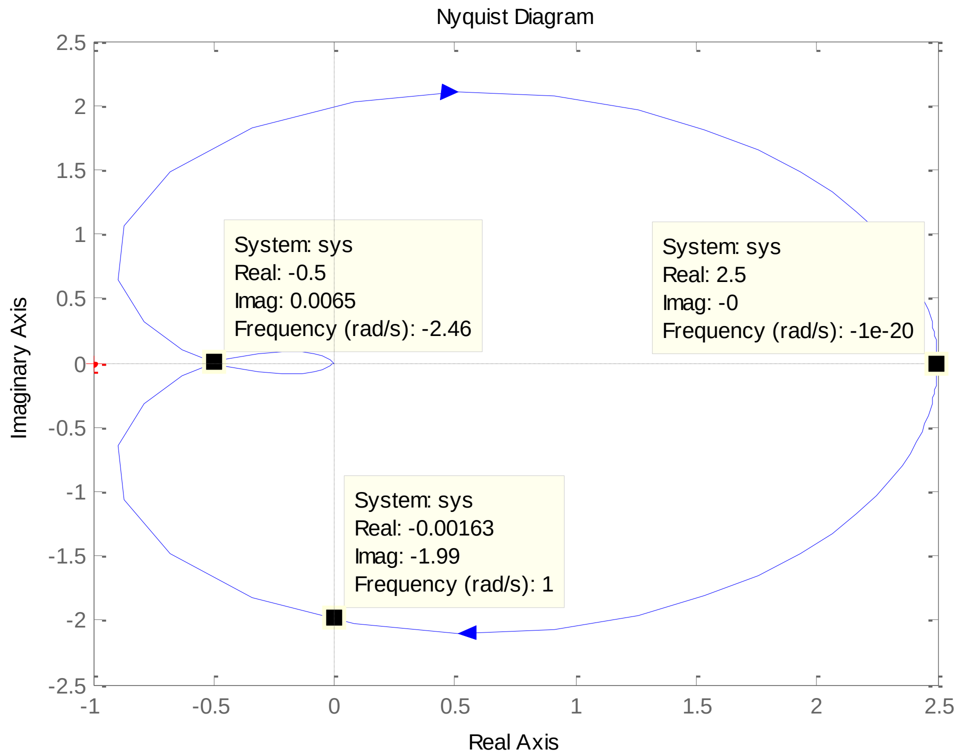
3- Find the table

ω	0	0.1	1	1.78	2.45	5	10	∞
Φ	0			-150	-180	-		-270
M	2.5		2	1	0.5			0
20logM	8		-6.02	0	6.02	-	-	0
Real **G(jω)H(jω)**	2.5		0		-0.5			0
Imag **G(jω)H(jω)**	0		-2		0			0

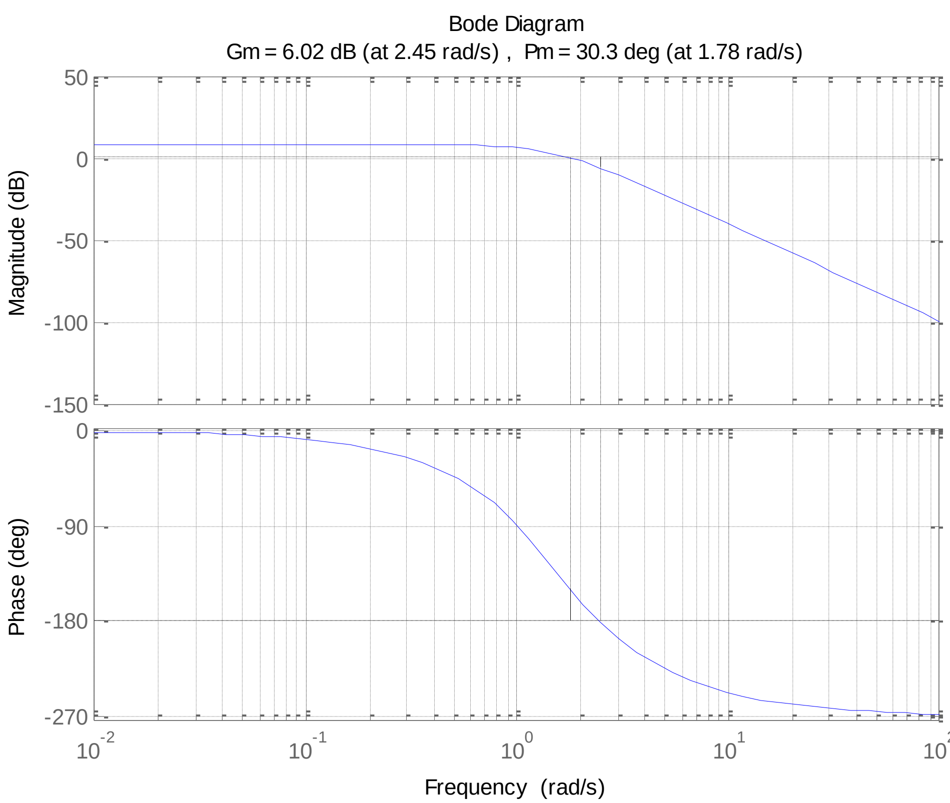
4- Plot the vector on the **jω − plane** where Φ in degrees as a straight line and determine M on this line
5- Plot the locus of the vector as points from the table
6- Find the gain and the phase margins from the plot

Prog. >>n=[10]; d=[1 4 6 4];

>> nyquist(n,d) >> margin(n,d) >> nichols(n,d)



Nyquist Diagram
System: sys
Real: -0.5
Imag: 0.0065
Frequency (rad/s): -2.46
System: sys
Real: 2.5
Imag: -0
Frequency (rad/s): -1e-20
System: sys
Real: -0.00163
Imag: -1.99
Frequency (rad/s): 1
Imaginary Axis
Real Axis



Bode Diagram
Gm = 6.02 dB (at 2.45 rad/s) , Pm = 30.3 deg (at 1.78 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)