

نموذج أسئلة و اجابة مادة ميكانيكا الموائع م 1112

تاريخ الامتحان الخميس 31 مايو 2018

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Benha University College of Engineering at Banha

Department of Mechanical Eng.

First Year Mechanical

Subject : Fluid Mechanics M1112

Questions For Final Examination Date May 31/2018

Examiner: Dr. Mohamed Elsharnoby Time: 180 min.

Solve the following five questions, and assume any missing data

1-a) Define the system, write and explain the following:

- Integral form of conservation of linear momentum.
- Energy Equation of fluid flow

b) Liquid enters a circular pipe of radius R with a linear velocity profile as a function of the radius with maximum velocity at the center equals U_{max} . After magical mixing, the velocity became parabolic Fig.1.

- (i) Write the equation which describes the velocity at the entrance.
- (ii) What is the ratio between the maximum velocities at inlet and at the exit (U_{max}/U)
- (iii) Calculate the momentum flux correction factor at the inlet of this flow.
- (iv) For $R=5\text{cm}$, and flow rate $Q=3\text{L/sec}$, momentum correction factor at exit $\beta=4/3$, find: U_{max} , U , and the force acting on the fluid between inlet and outlet.

2-a) Given is steady isothermal flow of water at 20°C through the device in Fig.2. Heat-transfer, gravity, and temperature effects are negligible. Known data are $D_1=9\text{ cm}$, $Q_1=220\text{ m}^3/\text{h}$, $P_1=150\text{ kPa}$, $D_2=7\text{ cm}$, $Q_2=100\text{ m}^3/\text{h}$, $P_2=225\text{ kPa}$, $D_3=4\text{ cm}$, and $P_3=265\text{ kPa}$. Compute the rate of shaft work done for this device and its direction.

2-b) A jet of water strikes the vertical plate in Fig.3, to maintain the water height h is constant, the feeding flow rate = 5 L/s , Assuming there are no losses in the nozzle ,find:

- i) The water height h in the tank.
- ii) The gauge pressure reading P .
- iii) The holding force F on the plate.

3-a) A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it (Fig. 4). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm . The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gage pressure at the center of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place.

b) For water flow between two plates the velocity profile is given by:

$$u(y) = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - yc) + \frac{(u_1 - u_0)}{c} y + u_0$$

Where c is the distance between the two plates, u_0 , u_1 are the speeds of the upper and lower plate respectively. For two stationary plates with 2m apart and pressure is decreasing by 10Pa each 1 km , kinematic viscosity of water ($\nu = 1.005 \times 10^{-6}\text{ m}^2/\text{s}$). Determine:

i) The velocity profile. ii) The flow rate between the two plates, and iii) the friction and the pressure coefficients.

c) Define the following: i) boundary layer, ii) fully developed flow, iii) entrance length L_e (write expressions for L_e in laminar and turbulent flows.

4-a)

In Fig.4, there are 25 m of 5-cm pipe, 250 m of 2.5-cm pipe, all roughness 0.046 mm the pump discharge 5 L/S of water at 20°C ($\nu = 1.005 \times 10^{-6} \text{ m}^2/\text{s}$), The difference in elevation of the two ponds is 100 m. calculate: the pump shaft horsepower (efficiency 82%), and the gauge pressure at points A, B.

b) Compute the displacement thickness, the momentum thickness, and the shape factor the local skin friction coefficient C_F assuming the velocity profile of laminar boundary layer is given by:

$$\frac{u}{u_o} = f\left(\frac{y}{\delta}\right) = f(\chi) = x - \frac{\chi^2}{2}$$

$$u = \begin{cases} u_o f(\chi) \rightarrow \text{for } y < \delta \\ u_o \rightarrow \text{for } y > \delta \end{cases}$$

5-a) Friction on the inside wall of a pipe. The shear stress τ_w on the pipe walls is a function of average fluid speed V , average wall roughness height ϵ , fluid density ρ , fluid viscosity μ , and inside pipe diameter D . Develop a nondimensional relationship between shear stress τ_w and the other parameters in the problem.

5-b) i) Redraw Fig.5-a and draw the characteristic curve of the three pumps connected in series.

ii) Redraw Fig.5-b and draw the characteristic curve of the three pumps connected in parallel.

(GOOD LUCK)



Figure 1

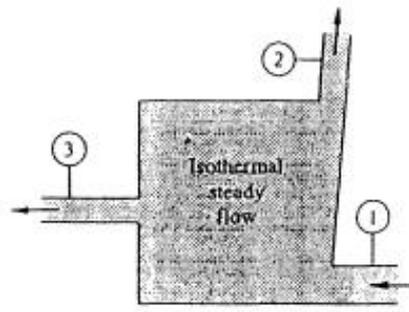


Figure 2

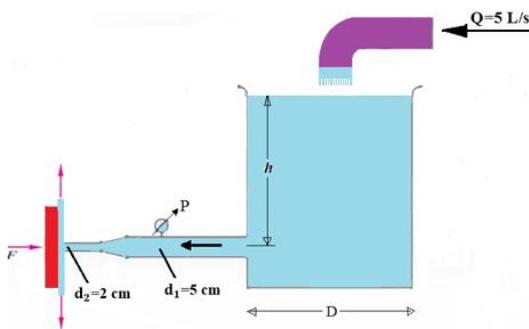


Figure 3

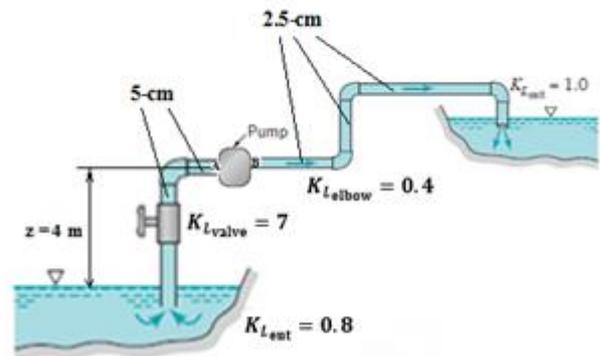


Figure 4

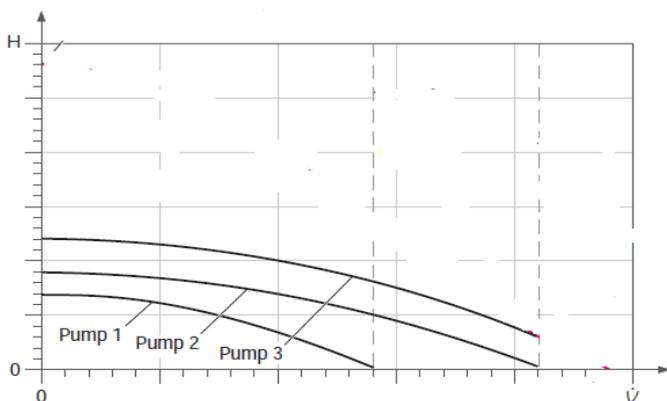


Fig.5-a

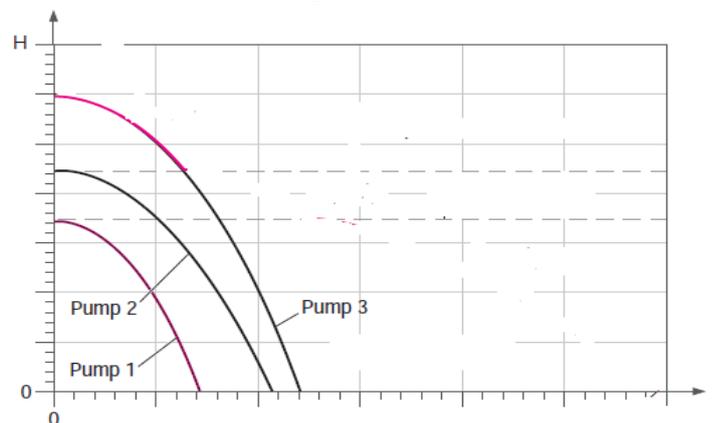


Fig.5-b

GOOD LUCK



Benha University College of Engineering at Banha
Department of Mechanical Eng.

Subject: **Fluid Mechanics**

Model Answer of the Final Exam Date: 31/5/2018

اجابة امتحان ميكانيكا الموائع م 1112 السنة الأولى ميكانيكا

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Conservation of linear momentum

which is a restatement of Newton's Second Law.

Newton's Second Law

○ In equation form this is written as:

$$\Sigma \underline{F}_{sys} = \frac{d}{dt} (m \underline{V})_{sys}$$

Where $m \underline{V}$ = the linear momentum of the system.

- Note that this is the same as Newton's second law, it is just written a little differently.
- Using the above equation, we can obtain a form of the equation in terms of acceleration:

$$\Sigma \underline{F}_{sys} = \frac{d}{dt} (m \underline{V})_{sys} = m \frac{d \underline{V}}{dt} + \underline{V} \frac{dm}{dt}$$

- This equation is obtained through expansion using the product rule.
- We know that ,

$$\frac{d \underline{V}}{dt} = \underline{a} \quad \text{and} \quad \frac{dm}{dt} = 0$$

By the conservation of mass

Using these ideas, we can then see that

$$\begin{aligned} \Sigma \underline{F}_{sys} &= m_{sys} \underline{a}_{sys} \\ \Sigma \vec{F}_{CV} &= \Sigma_{out} \dot{m} \vec{V} - \Sigma_{in} \dot{m} \vec{V} \\ \Sigma F_x &= \Sigma_{out} \dot{m} u - \Sigma_{in} \dot{m} u \\ \Sigma F_y &= \Sigma_{out} \dot{m} v - \Sigma_{in} \dot{m} v \\ \Sigma F_z &= \Sigma_{out} \dot{m} w - \Sigma_{in} \dot{m} w \end{aligned}$$

$$\sum \vec{F}_{CV} = \sum \vec{F}_{body} + \sum \vec{F}_{press} + \sum \vec{F}_{visc} + \sum \vec{F}_{other} = \sum_{out} \dot{m} \vec{V} - \sum_{in} \dot{m} \vec{V}$$

Conservation of Energy

○ For this, use the First Law of Thermodynamics in rate form to obtain the following equation:

$$\frac{dE_{sys}}{dt} = \dot{Q}_{sys} - \dot{W}_{sys}$$

○ Where E = the total energy of the system. In the above equation

$$\frac{dE_{sys}}{dt}$$

is the rate of change of system energy.

○ \dot{Q}_{sys} is the rate of heat added *to* the system $\left(\frac{\delta Q}{dt} \right)$

○ \dot{W}_{sys} is the rate of work done *by* the system $\left(\frac{\delta W}{dt} \right)$.

Because work is done by the system, the negative sign is in the equation for the first law of thermodynamics.

○ Now, these conservation laws must always hold for a system.

• Just as we did in the momentum equation, we can put this correction factor into the energy equation, and then treat all inlets and outlets as though they were one-dimensional, with average velocity V_{av} . With this correction, the general steady-state, steady flow (SSSF) form of the energy equation for a fixed control volume becomes:

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \sum_{out} \left(\tilde{h} + \frac{1}{2} \alpha V_{av}^2 + gz \right) \dot{m} - \sum_{in} \left(\tilde{h} + \frac{1}{2} \alpha V_{av}^2 + gz \right) \dot{m}$$

1-b

i) The velocity profile is linear with radius. Additionally, later a discussion on (i) relationship between velocity at interface to solid also referred as the (no) slip condition will be provided. This assumption is good for most cases with very few exceptions. It will be assumed that the velocity at the interface is zero. Thus, the boundary condition is



Figure 1

$U(r=R) = 0$ and $U(r=0) = U_{max}$ Therefore the velocity profile is

$$U(r) = U_{max} \left(1 - \frac{r}{R}\right)$$

ii) Where R is radius and r is the working radius (for the integration). The magical averaged velocity is obtained using the equation. For which

$$\int_0^R U_{max} \left(1 - \frac{r}{R}\right) 2\pi r dr = U_{ave} \pi R^2$$

The integration of the equation gives

$$U_{max} \pi \frac{R^2}{3} = U_{ave} \pi R^2 \Rightarrow U_{ave} = \frac{U_{max}}{3}$$

$$\text{The mean velocity at the outlet} = U_{ave} = \frac{U}{2} = \frac{U_{max}}{3} \rightarrow U_{max} = \frac{3}{2} U$$

iii) Calculating the momentum flux correction factor

$$\beta = \frac{1}{A} \int \left(\frac{U}{U_{ave}}\right)^2 dA = \frac{1}{\pi R^2} \int_0^R 3^2 \left(1 - \frac{r}{R}\right)^2 2\pi r dr$$

$$\beta = \frac{9 \times 2\pi}{\pi R^2} \int_0^R \left(1 - \frac{2r}{R} + \frac{r^2}{R^2}\right) r dr$$

$$\beta = \frac{18\pi}{\pi R^2} \left(\frac{r^2}{2} - \frac{2r^3}{3R} + \frac{r^4}{4R^2}\right)_0^R = \frac{18\pi}{\pi R^2} \left(\frac{R^2}{12}\right) = 1.5$$

iv) The mean velocity

$$U_{ave} = \frac{Q}{A} = \frac{3000}{\pi R^2} = \frac{120}{\pi} = 0.4247 \text{ m/sec} = \frac{U}{2} = \frac{U_{max}}{3} \rightarrow U_{max} = 1.274 \text{ m/sec}, U = 0.849 \text{ m/sec}$$

$$F = \left(\dot{m} V \beta\right)_{out} - \left(\dot{m} V \beta\right)_{in} = 5 \times 0.4247 \times 1.5 - 5 \times 0.4247 \times \frac{4}{3} = 0.3533 \text{ N}$$

2-a) For continuity, $Q_3 = Q_1 - Q_2 = 120 \text{ m}^3/\text{hr}$. Establish the velocities at each port:

$$V_1 = \frac{Q_1}{A_1} = \frac{220/3600}{\pi(0.045)^2} = 9.61 \frac{\text{m}}{\text{s}}; \quad V_2 = \frac{100/3600}{\pi(0.035)^2} = 7.22 \frac{\text{m}}{\text{s}}; \quad V_3 = \frac{120/3600}{\pi(0.02)^2} = 26.5 \frac{\text{m}}{\text{s}}$$

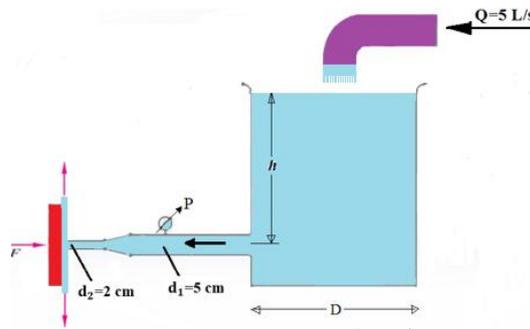
With gravity and heat transfer and internal energy neglected, the energy equation becomes

$$\dot{Q} - \dot{W}_s - \dot{W}_v = \dot{m}_3 \left(\frac{p_3}{\rho_3} + \frac{V_3^2}{2} \right) + \dot{m}_2 \left(\frac{p_2}{\rho_2} + \frac{V_2^2}{2} \right) - \dot{m}_1 \left(\frac{p_1}{\rho_1} + \frac{V_1^2}{2} \right),$$

$$\text{or: } -\dot{W}_s/\rho = \frac{100}{3600} \left[\frac{225000}{998} + \frac{(7.22)^2}{2} \right] + \frac{120}{3600} \left[\frac{265000}{998} + \frac{(26.5)^2}{2} \right] + \frac{220}{3600} \left[\frac{150000}{998} + \frac{(9.61)^2}{2} \right]$$

Solve for the shaft work: $\dot{W}_s = 998(-6.99 - 20.56 + 12.00) \text{ H} = \mathbf{15500 \text{ W Ans.}}$
(negative denotes work done *on* the fluid)

2-b)



Applying the continuity equation for constant h

$$U_2 = \frac{Q}{A_2} = \frac{5000}{\pi(1)^2} = 15.92 \text{ m/sec}$$

$$\text{For no losses } U_2 = \sqrt{2gh} \rightarrow h = 12.93 \text{ m}$$

$$U_1 = \frac{Q}{A_1} = \frac{5000}{\pi\left(\frac{5}{2}\right)^2} = 2.5472 \text{ m/sec}$$

Applying Bernoulli Eqn. between 1 and 2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \Rightarrow 0 + 12.93 + 0 = \frac{p_2}{\rho g} + \frac{(2.5472)^2}{2g} \Rightarrow p_2 = 123.545 \text{ kPa}$$

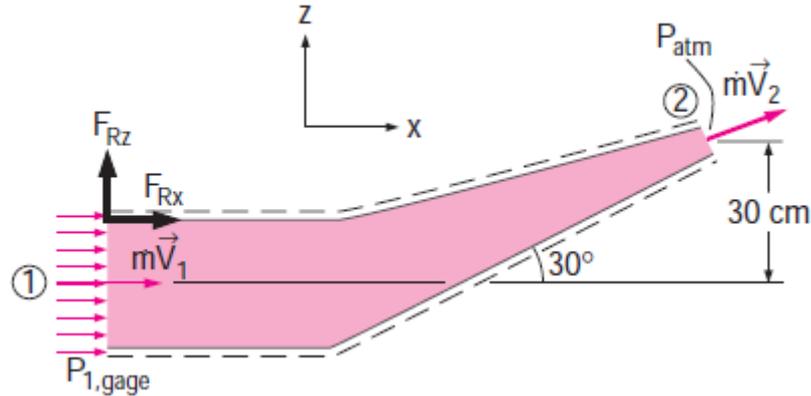
The force acting on the plate:

$$F = \dot{m}(U_{\text{out}} - U_{\text{in}}) = 5 \times 12.93 = 64.65 \text{ N}$$

3-a A reducing elbow deflects water upward and discharges it to the atmosphere. The pressure at the inlet of the elbow and the force needed to hold the elbow in place are to be determined.

Assumptions 1 The flow is steady, and the frictional effects are negligible.

2 The weight of the elbow and the water in it is negligible. 3 The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. 4 The flow is turbulent and fully developed at both the inlet and outlet of the control volume, and we take the momentum-flux correction factor to be $\beta=1.03$.



Properties We take the density of water to be 1000 kg/m^3 . **Analysis** (a) We take the elbow as the control volume and designate the inlet by 1 and the outlet by 2. We also take the x - and z -coordinates as shown. The continuity equation for this one-inlet, one-outlet, and steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 14 \text{ kg/s}$. Noting that $\dot{m} = \rho VA$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0113 \text{ m}^2)} = 1.24 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(7 \times 10^{-4} \text{ m}^2)} = 20.0 \text{ m/s}$$

We use the Bernoulli equation as a first approximation to calculate the pressure. Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{\text{atm}}$, the Bernoulli equation for a streamline going through the center of the elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$P_1 - P_2 = \rho g \left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1 \right)$$

$$P_1 - P_{\text{atm}} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \times \left(\frac{(20 \text{ m/s})^2 - (1.24 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0.3 - 0 \right) \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$P_{1, \text{gage}} = 202.2 \text{ kN/m}^2 = \mathbf{202.2 \text{ kPa}} \quad (\text{gage})$$

(b) The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

We let the x - and z -components of the anchoring force of the elbow be F_{Rx} and F_{Rz} , and assume them to be in the positive direction. We also use gage pressure since the atmospheric pressure acts on the entire control surface. Then the momentum equations along the x - and z -axes become

$$F_{Rx} + P_{1, \text{gage}} A_1 = \beta \dot{m} V_2 \cos \theta - \beta \dot{m} V_1$$

$$F_{Rz} = \beta \dot{m} V_2 \sin \theta$$

Solving for F_{Rx} and F_{Rz} , and substituting the given values,

$$\begin{aligned} F_{Rx} &= \beta \dot{m} (V_2 \cos \theta - V_1) - P_{1, \text{gage}} A_1 \\ &= 1.03(14 \text{ kg/s})[(20 \cos 30^\circ - 1.24) \text{ m/s}] \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \\ &\quad - (202,200 \text{ N/m}^2)(0.0113 \text{ m}^2) \\ &= 232 - 2285 = \mathbf{-2053 \text{ N}} \end{aligned}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin \theta = (1.03)(14 \text{ kg/s})(20 \sin 30^\circ \text{ m/s}) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = \mathbf{144 \text{ N}}$$

The negative result for F_{Rx} indicates that the assumed direction is wrong, and it should be reversed. Therefore, F_{Rx} acts in the negative x -direction.

3-b)

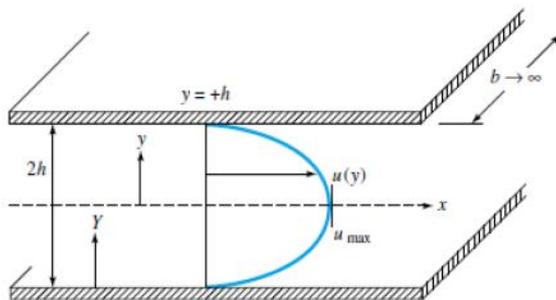
In this case $u_0 = u_1 = 0$. The velocity expression then becomes:

$$u(y) = -\frac{(P_0 - P_1)}{2\mu l}(y^2 - yc)$$

i) The velocity profile. ii) The flow rate between the two plates, and iii) the friction and the pressure

$$u(y) = \frac{-10}{2 \times 1000 \times 1.005 \times 10^{-6} \times 1000}(y^2 - 2y) = \frac{-5}{1.005}(y^2 - 2y) \text{ m/sec}$$

The velocity profile across the two plates is therefore parabolic, as shown below.



The volume flow rate between the plates per unit width is :

$$Q = \int_0^c u(y)dy = \frac{-5}{1.005 \times 0} \int_0^2 (y^2 - 2y)dy = \frac{-5}{1.005} \left(\frac{y^3}{3} - y^2 \right) = \frac{5}{1.005} \times \frac{4}{3} = 6.633499 \text{ m}^3 / \text{m sec}$$

The mean velocity = $6.633499 / 2 = 3.31675 \text{ m/sec}$

$$\tau = \mu \left. \frac{du}{dy} \right|_{y=0} = 1000 \times 1.005 \times 10^{-6} \times \frac{10}{1.005} = 0.01 \text{ N/m}^2$$

$$\Delta p \times c = \tau \times l \rightarrow \Delta p \times 2 = \tau \times 1000 \rightarrow \Delta p = 5 \text{ N/m}^2$$

$$Cp = \frac{5}{0.5 \times 1000 \times (3.31675)^2} = 0.00090909$$

$$C_f = 0.00000181818$$

3-c

i) This region, where there is a velocity profile in the flow due to the shear stress at the wall, we call the **boundary layer**.

Boundary layer is the region near a solid where the fluid motion is affected by the solid boundary.

ii) Once the boundary layer has reached the centre of the pipe the flow is said to be **fully developed**. (Note that at this point the whole of the fluid is now affected by the boundary friction.)

At a finite distance from the entrance, the boundary layers merge and the inviscid core disappears. The flow is then entirely viscous, and the axial velocity adjusts slightly further until at $x = L_e$ it no longer changes with x and is said to be fully developed, $v = v(r)$ only.

iii) The length of pipe before fully developed flow is achieved is different for the two types of flow. The length is known as the entry length.

The entrance length L_e is estimated for laminar flow to be :

$$L_e/D = 0.06 Re_D \text{ for laminar}$$

$$L_e/D = 4.4 Re_D^{1/6} \text{ for turbulent flow}$$

Where L_e is the entrance length; and

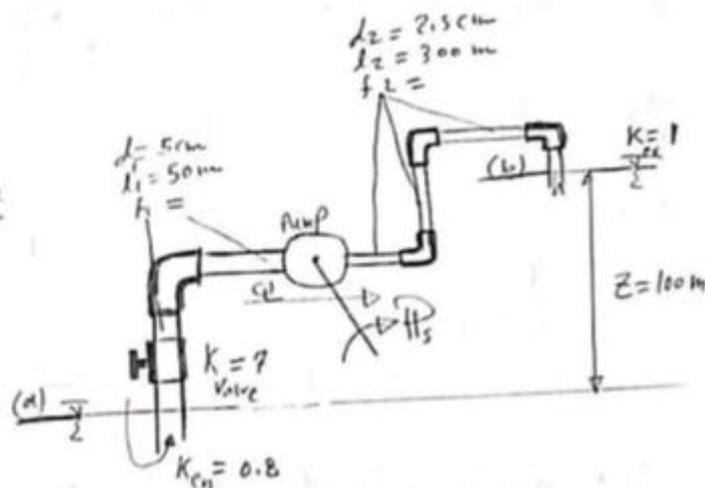
Re_D is the Reynolds number based on Diameter

4-a)

$$Q = 0.01 \frac{m^3}{s}$$

$$\nu = 1005 \times 10^{-6} \frac{m^2}{s}$$

$$\epsilon = 0.046 \text{ mm}$$



applying energy eq Let (a-b) \Rightarrow

$$\frac{P_a}{\rho} + \frac{V_a^2}{2g} + z_a = \frac{P_b}{\rho} + \frac{V_b^2}{2g} + z_b + \sum h_{L_{a-b}} - h_p$$

$$h_p = 100 + \sum h_{L_{a-b}}$$

$\sum \frac{f_i L_i}{d_i} \frac{V_i^2}{2g}$ (major)

 $\sum K_i \frac{V_i^2}{2g}$ (minor)

$$h_p = 100 + \left(\frac{f_1 l_1}{d_1} + K_{\text{int}} + K_{\text{valve}} + K_{\text{elbow}} \right) \frac{V_1^2}{2g} + \left(\frac{f_2 l_2}{d_2} + 3K_{\text{elbow}} + K_{\text{exit}} \right) \frac{V_2^2}{2g} \Rightarrow$$

$$\therefore V_1 = \frac{Q}{\frac{\pi}{4} d_1^2} = \frac{0.01}{\frac{\pi}{4} (0.05)^2} = 5.1 \text{ m/s}$$

$$V_2 = \frac{Q}{\frac{\pi}{4} d_2^2} = \frac{0.01}{\frac{\pi}{4} (0.025)^2} = 20.37 \text{ m/s}$$

4-b)

b) Compute the displacement thickness, the momentum thickness, and the shape factor the local skin friction coefficient C_F assuming the velocity profile of laminar boundary layer is given by:

$$\frac{u}{u_\infty} = f\left(\frac{y}{\delta}\right) = f(\chi) = x - \frac{\chi^2}{2}$$

$$u = \begin{cases} u_\infty f(\chi) \rightarrow \text{for } y < \delta \\ u_\infty \rightarrow \text{for } y > \delta \end{cases}$$

The displacement thickness is given by :

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy = \int_0^\delta \left(1 - f\left(\frac{y}{\delta}\right)\right) dy = \int_0^1 \left(1 - x + \frac{\chi^2}{2}\right) d\chi = \delta \frac{2}{3}$$

The momentum thickness θ

$$\theta = \int_0^\infty \frac{u}{u_\infty} \left(1 - \frac{u}{u_\infty}\right) dy = \int_0^\delta f\left(\frac{y}{\delta}\right) \left(1 - f\left(\frac{y}{\delta}\right)\right) dy = \int_0^1 \left(x - \frac{\chi^2}{2}\right) \left(1 - x + \frac{\chi^2}{2}\right) d\chi =$$

$$\delta \int_0^1 \left(x - \frac{3}{2}x^2 + x^3 - \frac{\chi^4}{4}\right) d\chi = \delta \left(\frac{1}{2} - \frac{1}{2} + \frac{1}{4} - \frac{1}{20}\right) = 0.2\delta$$

$$\text{The shape factor } H = \frac{\delta^*}{\theta} = \frac{2}{3} \times 5 = \frac{10}{3} = 3.3333$$

5-a)

Assumptions 1 The flow is fully developed. 2 The fluid is incompressible. 3 No other parameters are significant in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters.

Step 1 There are six variables and constants in this problem; $n = 6$. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants:

List of relevant parameters:

Step 2 The primary dimensions of each parameter are listed. Note that shear stress is a force per unit area, and thus has the same dimensions as pressure.

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (M, L, and t).

Reduction: $j=3$

If this value of j is correct, the expected number of Π 's is $k = n - j = 6 - 3 = 3$

Step 4 We choose three repeating parameters since $j=3$. Following the guide lines, we cannot pick the dependent variable τ_w . We cannot choose both ϵ , and D since their dimensions are identical, and it would not be desirable to have μ or ϵ , appear in all the Π 's. The best choice of repeating parameters is thus V , D , and ρ .

$$\Pi_1 = \tau_w V^{a_1} D^{b_1} \rho^{c_1} \rightarrow \{\Pi_1\} = \{(m^1 L^{-1} t^{-2})(L^1 t^{-1})^{a_1} (L^1)^{b_1} (m^1 L^{-3})^{c_1}\}$$

from which $a_1 = -2$, $b_1 = 0$, and $c_1 = -1$, and thus the dependent Π is

$$\Pi_1 = \frac{\tau_w}{\rho V^2}$$

Similarly, the two independent Π 's are generated, the details of which are left for the reader:

$$\Pi_2 = \mu V^{a_2} D^{b_2} \rho^{c_2} \rightarrow \Pi_2 = \frac{\rho V D}{\mu} = \text{Reynolds number} = \text{Re}$$

$$\Pi_3 = \epsilon V^{a_3} D^{b_3} \rho^{c_3} \rightarrow \Pi_3 = \frac{\epsilon}{D} = \text{Roughness ratio}$$

Step 6 We write the final functional relationship as

$$f = \frac{8\tau_w}{\rho V^2} = f\left(\text{Re}, \frac{\epsilon}{D}\right)$$

5-b)i

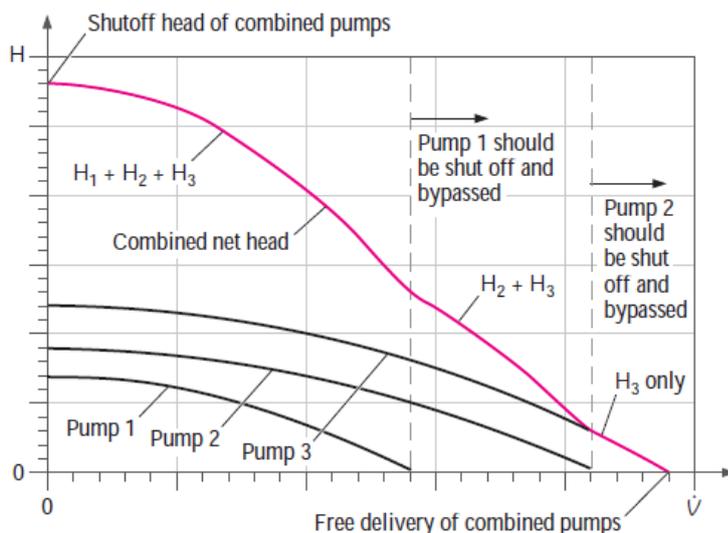


FIGURE 14-23

Pump performance curve (dark blue) for three dissimilar pumps in series. At low values of volume flow rate, the combined net head is equal to the sum of the net head of each pump by itself. However, to avoid pump damage and loss of combined net head, any individual pump should be shut off and bypassed at flow rates larger than that pump's free delivery, as indicated by the vertical dashed gray lines. If the three pumps were identical, it would not be necessary to turn off any of the pumps, since the free delivery of each pump would occur at the same volume flow rate.

5-b)ii

Pump performance curve (dark blue) for three pumps in parallel. At a low value of net head, the combined capacity is equal to the sum of the capacity of each pump by itself. However, to avoid pump damage and loss of combined capacity, any individual pump should be shut off at net heads larger than that pump's shutoff head, as indicated by the horizontal dashed gray lines. That pump's branch should also be blocked with a valve to avoid reverse flow. If the three pumps were identical, it would not be necessary to turn off any of the pumps, since the shutoff head of each pump would occur at the same net head.

