نموذج الأسئلة و الاجابة لمادة : التحكم الآلى م 1352

التاريخ السبت 19 مايو 2018

أستاذ المادة : د. محمد عبد اللطيف الشرنوبى

Benha University
College of Engineering at Benha
Mechanical Eng. Dept. **3rd Year Mechanics**
Subject :Automatic Control M1352 **Date:19/5/2018**

Questions For Final Examination

Examiner : Dr. Mohamed Elsharnoby **Time :180 min.**

1-Given the block diagram in figure 1, where r(t) is a unit step function.

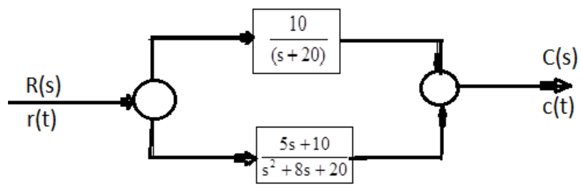

Figure 1

i) Find the steady state value of c(t).
ii) Find the nymerical value of the settling time for 2%tolerence.
iii) Choose: the response will be mainly: overdamped, critically damped, or underdamped? Justify your answer.

1-b)Using Masson's gain formula obtain the closed-loop transfer function of the system whose signal flow graph is shown in figure 2.

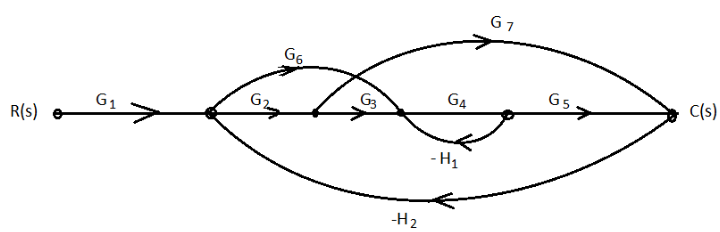

Figure 2

1-c)

If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{S^2 + 5S + 6}{S^4 + 2S^3 + 4S^2 + 3S}$$

i) Draw a signal flow graph represents this system.
ii) Deduce the state space representation of the system.
iii) Write the differential equation of the system in canonical form.

2-a) Given the block diagram for a control system in figure 3, $G(s) = \frac{10}{(s+2)(s+10)}$

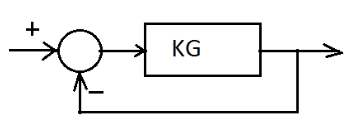

Figure 3

i) Find K such that the steady state unit step error is 0.1.

ii) Find K such that $\zeta = \frac{1}{\sqrt{2}}$.

2-b) The system shown in figure 4 has parameters ζ=0.4 and ω_n=5 rad/sec. The system is subjected to a unit step input, find the resulting rise time (t_r), peak time (t_p), settling time (t_s) and maximum overshoot (M_p).

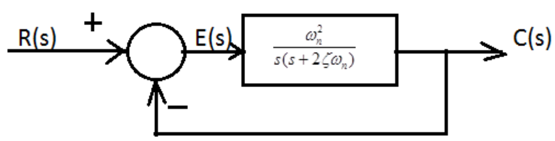

Figure 4

3-a) Consider the standard feedback system shown in figure 5 with $G_p(s) = \frac{(1-s)}{(1+s)^2}$

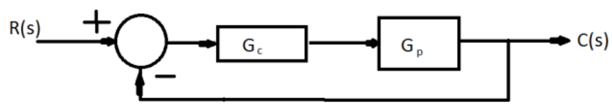

Figure 5

We use a proportional controller $G_c(s) = K$, with K >0.

(i) Determine the range of K for which the feedback system is stable.

(ii) Let $k = \sqrt{2}$, calculate the gain and phase margins.

(iii) Let $k = \sqrt{3}$, and sketch the Nyquist plot (calculate real axis crossings of $G(i\omega)$)

3-b) Consider a unity gain feedback control system. The plant transfer function is G(s)=1/(s^2+5s+6). Let the controller be of the form C(s) =K(s+z)/(s+p). Design the controller (ie choose K, z, p>0) so that the closed loop system has poles at $-1 \pm j$.

4-a) Sketch the root locus for the system given in figure 3 with

$$G_c(s) = K\frac{(s+1)}{s} \qquad G_p(s) = \frac{1}{s(s+7)^2}$$

Make sure you provide verbal description on the following: open-loop pole/zero map; real axis decision; and asymptotes. Find the jw-axis crossing point and the corresponding value of K.

4-b) Consider the following system where G(s) is a transfer function. Asymptotic Bode plots of G(s) is given in figure 6 below. For calculations, you may use these Asymptotic plots.

i) Find the gain margin of the system.

ii) Find the phase margin of the system.

iii) Find the transfer function of the system. .

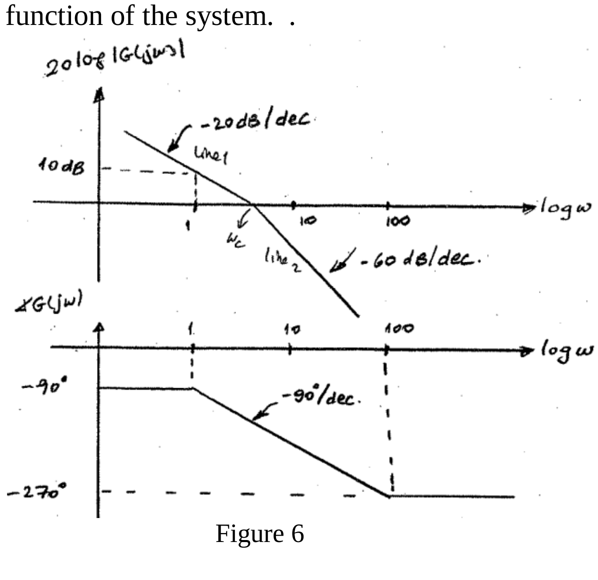

Figure 6

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1-a) The transfer function is $T(s)=\dfrac{10}{s+20}+\dfrac{5s+10}{s^2+8s+20}=\dfrac{C(s)}{R(s)}$

For unit step input $R(s)=\dfrac{1}{s}\rightarrow\therefore C(s)=\dfrac{1}{s}\left(\dfrac{10}{s+20}+\dfrac{5s+10}{s^2+8s+20}\right)$

i) The steady state value $C(\infty)=\lim_{s\to 0} sC(s)=\lim_{s\to 0}\left(\dfrac{10}{s+20}+\dfrac{5s+10}{s^2+8s+20}\right)=1$

ii) From the second order part of the transfer function $\dfrac{5s+10}{s^2+8s+20}, 2\zeta\omega_n=8, \omega_n^2=20$

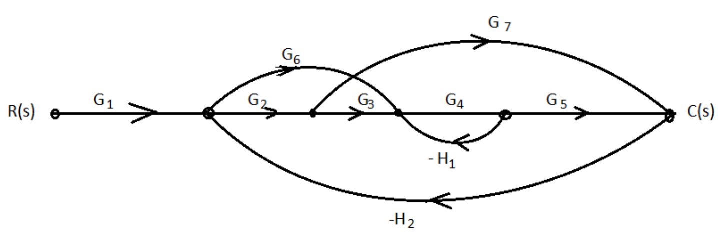
$$\zeta=\frac{4}{\sqrt{20}}=0.894, \omega_n=\sqrt{20} rad/\sec$$

$$t_s=\frac{4}{\zeta\omega_n}=1\sec$$

iii)

$0\prec\zeta\prec 1$ under damped

1-b)



Listing of loops and paths

Loops

$L_1=-G_4H_1$, $L_2=-G_2G_3G_4G_5H_2$, $L_3=-G_2G_7H_2$, $L_4=-G_6G_4G_5H_2$

Paths

$M_1=G_1G_2G_3G_4G_5$, $M_2=G_1G_6G_4G_5$, $M_1=G_1G_2G_7$

$\Delta_1=1$, $\Delta_2=1$, $\Delta_3=1+G4H_1$

$\Delta=1+G_4H_1+G_2G_3G_4G_5H_2+G_2G_7H_2+G_6G_4G_5H_2+G_4H_1\, G_2G_7H_2$

$$T(s)=\frac{\sum \Delta_i M_i}{\Delta}$$

1-c)

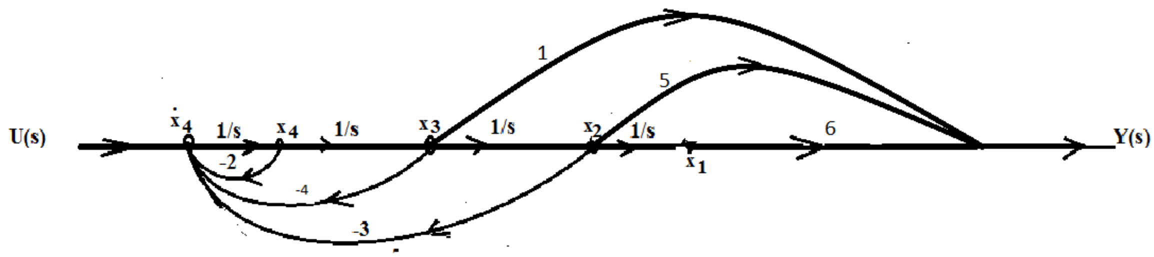
If the transfer function of a system is given by:

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 5s + 6}{s^4 + 2s^3 + 4s^2 + 3s}$$

Devide both numenator and denomenator by $\mathbf{s^6}$

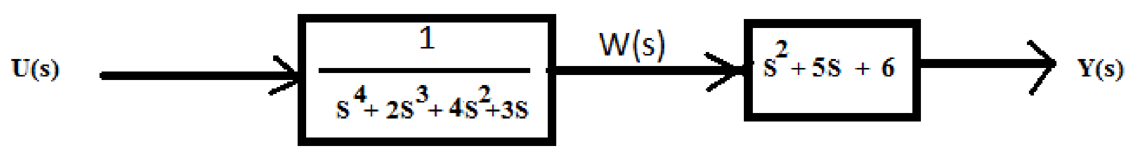
we get $$TF = \frac{\frac{1}{s^2} + \frac{5}{s^3} + \frac{6}{s^4}}{1 + \frac{2}{s} + \frac{4}{s^2} + \frac{3}{s^3}}$$

i) signal flow graph represents this system



Ii)Deduce the state space representation of the system.,

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 5s + 6}{s^4 + 2s^3 + 4s^2 + 3s}$$



$$\frac{W(s)}{U(s)} = \frac{1}{s^4 + 2s^3 + 4s^2 + 3s}$$

$$U(s) = (s^4 + 2s^3 + 4s^2 + 3s)W(s)$$

$$u(t) = \ddddot{w} + 2\dddot{w} + 4\ddot{w} + 3\dot{w}$$

$$\ddddot{w} = u - 2\dddot{w} - 4\ddot{w} - 3\dot{w}$$

$$\ddddot{w} = \dot{x}_4, \dot{w} = x_{1,} w = x_2 = \dot{x}_1, \ddot{w} = x_3 = \dot{x}_2, \dddot{w} = x_4 = \dot{x}_3, \ \dddot{w} = x_4 = \dot{x}_3$$

$$\dot{x}_4 = u - 2x_4 - 4x_3 - 2x_2$$

$$\dot{X} = AX+BU = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -3 & -4 & -2 \end{bmatrix} \begin{bmatrix} X \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = CX + DU = \begin{bmatrix} 6 & 5 & 1 & 0 \end{bmatrix} X + 0.U$$

2-a) i) Find K such that the steady state unit step error is 0.1.

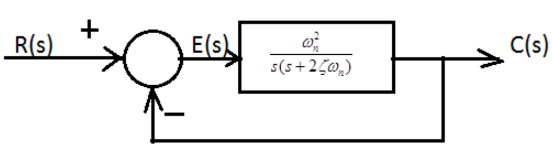
The steady state error for unit step $e_{ss} = \dfrac{1}{1+k_p} = 0.1 \rightarrow k_p = 9$

$$k_p = \lim_{s\rightarrow 0} \frac{10k}{(s+2)(s+10)} = 9 \quad \rightarrow \therefore k = 18.$$

ii) The denominator of the closed loop transfer is given by $s^2 + 12s + 10k + 20$

$$2\zeta\omega_n = 12 \rightarrow \omega_n = 6\sqrt{2} \rightarrow \omega_n^2 = 10k + 20 = 72 \rightarrow \therefore k = 5.2$$

2-b)) The system shown in figure 4 has parameters ζ=0.4 and ω_n=5 rad/sec. The system is subjected to a unit step input, find the resulting rise time (t_r), peak time (t_p), settling time (t_s) and maximum overshoot (M_p).



ii) $t_r \Longrightarrow t_r = \dfrac{\pi - \theta}{\omega_d}, \theta = \cos^{-1}\zeta$ = ,t_r=0.4 sec 0—100%

t_r=0.36 sec for 10%---100%

$t_p \Longrightarrow t_p = \pi / \omega_d = \pi / \omega_n \sqrt{1-\zeta^2}$ = 0.6856 se,

$t_s = \Longrightarrow t_s \cong \dfrac{4}{\sigma} = 4T$ = 2 sec,

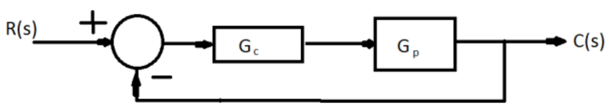
t_d = ,

$M_P = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$ =25.38%

$$t_s \le \frac{3 - 0.5\ln(1-\zeta^2)}{\zeta\omega_n} \quad \text{Or} \quad t_s \cong \frac{3}{\zeta\omega_n}$$

$$\text{MPO} = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\%$$

3-a $G_p(s) = \dfrac{(1-s)}{(1+s)^2}$



For proportional controller G_c =k

Using routh criterion

s^2	1	(1+k)
s^1	2-k	
s^0	1+k	

K < 2, K >- 1 i.e 0 < K < 2

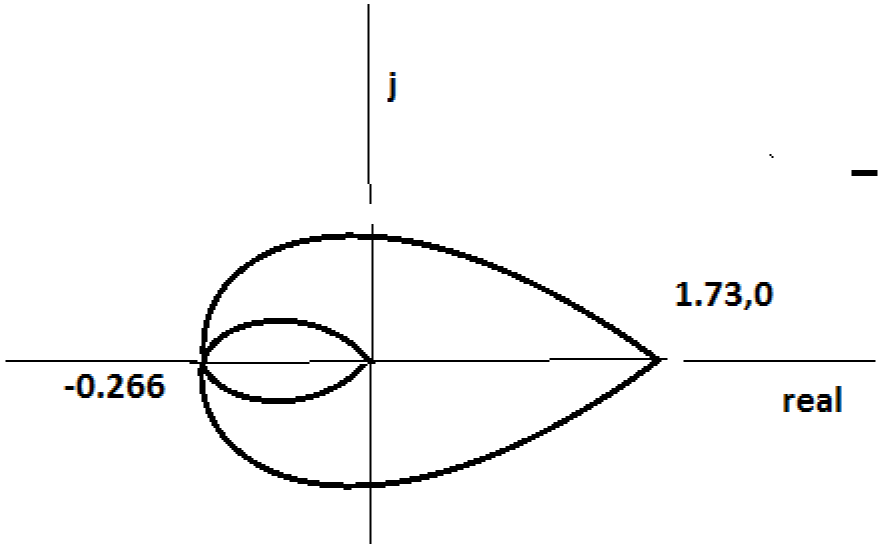
iii) For $k = \sqrt{2}, G(s) = \dfrac{\sqrt{2}(1-s)}{(1+s)^2}$, for phase 180

$\omega = \pm\sqrt{3}$ rad/sec

GM=4.771 db

NFor $|G| = 1.0$ PM = 180-135 = 45 deg.

For $k = \sqrt{3}, G(s) = \dfrac{\sqrt{3}(1-s)}{(1+s)^2}$



Nyquist plot

iv) $\Rightarrow t_r = \dfrac{\pi - \theta}{\omega_d} 1$

4-a)

i) Open loop poles are at 0 ,0 , -7, -7.

ii) Open loop zero at -1.

iii) Number of asymptotes = 4 – 1 = 3.

iv) Angles of asymptotes with real axis are 60^o ,180^o, 300^o.

v) Intersection point of asymptotes on the real line = (-7-7+1)3= -13/3

vi) Intersection with the imaginary axis

The characteristic equation is given by :

$S^2(S+7)^2 + K(S+1) = 0$

$S^4+4S^3+49S^2+kS+K=0$

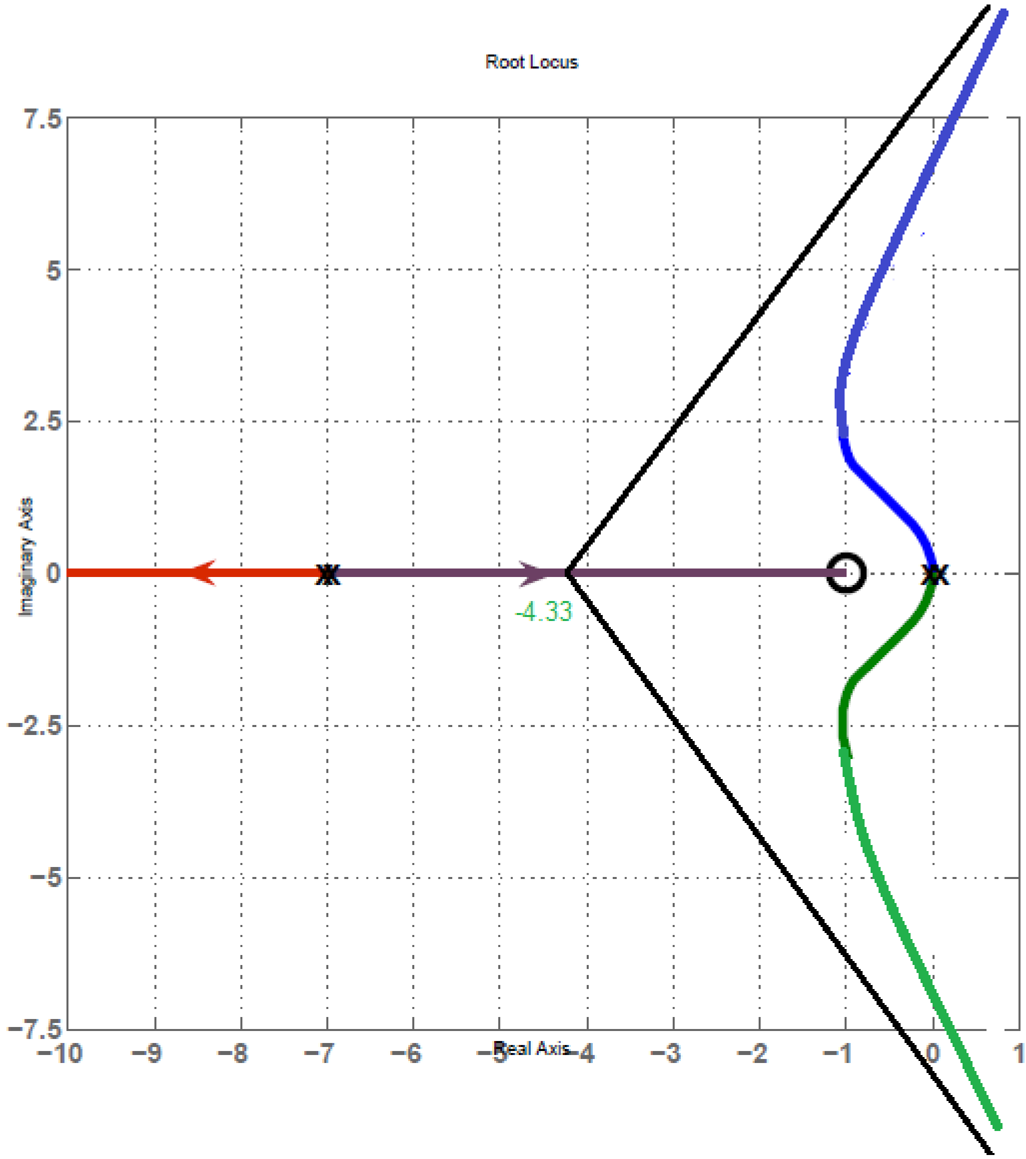
Put $S = j\omega$

$\omega^4 -4j\omega^3-49\omega^2+jk\omega +k=0$

$-4\omega^3+k\omega = 0$ $\qquad$ $k=4\omega^2$

$\omega^4 -49\omega^2+k=0$

$\omega^4 -45\omega^2=0$ $\quad$ $\omega=0.$ $\quad$ $\omega=\pm (45)^{1/2}$ $\quad$, K = 180

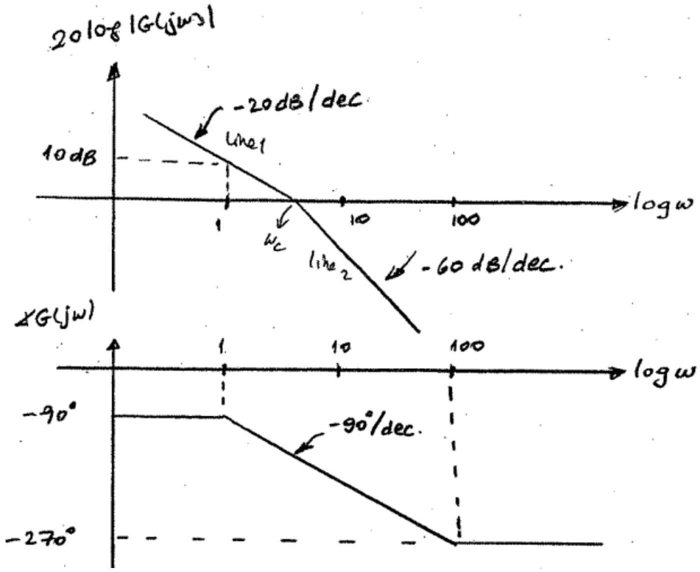


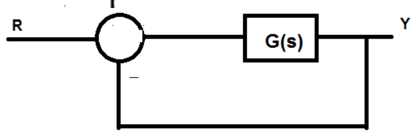
4-b) Consider the following system where G(s) is a transfer function. Asymptotic Bode plots of G(s) is given in figure 5 below. For calculations, you may use these Asymptotic plots.

iii) Find the gain margin of the system.

iv) Find the phase margin of the system.

v) Find the transfer function of the system. .





line 1 $\Rightarrow$ $20\log K - 20\log w$ $\Rightarrow$ $w=1 \Rightarrow 10\,dB \Rightarrow 20\log K = 10 \Rightarrow$ $\boxed{K=\sqrt{10}}$

$\Rightarrow$ $20\log K - 20\log w_c = 0 \Rightarrow$ $\boxed{w_c = K = \sqrt{10}}$

line 2 $\Rightarrow$ $A - 60\log w$ $= 0$ at $w_c = \sqrt{10} \Rightarrow$ $A - 60\log\sqrt{10} = 0 \Rightarrow A = 30\ dB.$ (05)

i) at $w_c = \sqrt{10}$ $\Rightarrow$ $\angle = -90 - 90\log w_c = -90 - 45 = -135°$ $\Rightarrow$ $\boxed{PM = 180 - 135 = 45°}$

ii) $-90 - 90\log w_o = -180 \Rightarrow$ $w_o = 10$ rad/sec. $\Rightarrow$ $30 - 60\log 10 = -30\,dB$ (05)

$\Rightarrow$ $\boxed{GM = 30\ dB}$

iii) $$G(s) = \frac{K}{s\left(\frac{s}{w_c}+1\right)^2} = \frac{\sqrt{10}}{s\left(\frac{s}{\sqrt{10}}+1\right)^2} = \frac{3.16}{s\,(0.32s+1)^2}$$ (05)

Name: Section: #

5-

*** GOOD LUCK ***