

## EXAM SOLUTION

### Solution of Question ①

$$I. \quad E_p = \text{zero} \quad E_1 = E_2 \quad \frac{K_e q_1 * R}{(R^2 + R^2)^{\frac{3}{2}}} = \frac{K_e q_2 * 3R}{((3R)^2 + R^2)^{\frac{3}{2}}}$$

$$\frac{q_2}{q_1} = \frac{1}{3} * \frac{10R^{\frac{3}{2}}}{2R^{\frac{3}{2}}} = 3.73$$

$$II. \quad a) \frac{q}{\frac{4}{3}\pi\frac{R^3}{27}} = \frac{Q}{\frac{4}{3}\pi R^3} \quad q = \frac{Q}{27}$$

$$b) E' = E \frac{r}{R} = \frac{E}{2}$$

$$III. \quad V_A - V_B = -E \cdot L = -(12)(-5) = 60 \text{V}$$

$$b) \frac{q_1}{q_2} = \frac{r_1}{r_2} \quad \frac{20 - q_2}{q_2} = \frac{4}{6} \quad q_2 = 12 \mu\text{C} \quad \text{and} \quad q_1 = 8 \mu\text{C}$$

$$V_1 = V_2 = \frac{K_e * q_1}{r_1} = 1800000 \text{V} = 1.8 \text{MV}$$

### Solution of Question ②

I.

$$\therefore C = \kappa C_o$$

$$\therefore C_o = \frac{2 \times 10^{-9}}{5} = 0.4 \text{ nF}$$

$$Q_o = Q = CV = 2 \times 10^{-9} \times 100 = 0.2 \mu\text{C}$$

$$\therefore V_o = \frac{Q_o}{C_o} = \frac{0.2 \times 10^{-6}}{0.4 \times 10^{-9}} = 500 \text{ V}$$

$$U_o = \frac{1}{2} C_o V_o^2 = 50 \mu\text{J}$$

II. We use  $I = nqAv_d$ , where  $n$  is the number of charge carriers per unit volume and is identical to the number of atoms per unit volume. We assume a contribution

of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms,  $N_A$ , has a mass of 27.0 g. Thus, the mass per atom is

$$m = \frac{27.0 \text{ g}}{N_A} = \frac{27.0 \text{ g}}{6.02 \times 10^{23}} = 4.49 \times 10^{-23} \text{ g/atom.}$$

Thus,

$$n = \frac{\rho}{m} = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70 \text{ g/cm}^3}{4.49 \times 10^{-23} \text{ g/atom}}$$

$$n = 6.02 \times 10^{22} \text{ atoms/cm}^3 = 6.02 \times 10^{28} \text{ atoms/m}^3$$

Therefore,

$$v_d = \frac{I}{nqA} = \frac{5.00 \text{ A}}{(6.02 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(4.00 \times 10^{-6} \text{ m}^2)}$$

$$= 1.30 \times 10^{-4} \text{ m/s}$$

$$\text{or, } v_d = \boxed{0.130 \text{ mm/s}}.$$

III.

$$r = \frac{m_e v}{eB} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.50 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^{-3} \text{ T})}$$

$$= 0.0427 \text{ m} = \boxed{4.27 \text{ cm}}$$

The time to complete one revolution around the orbit (i.e., the period) is

$$T = \frac{\text{distance traveled}}{\text{constant speed}} = \frac{2\pi r}{v} = \frac{2\pi(0.0427 \text{ m})}{1.50 \times 10^7 \text{ m/s}} = \boxed{1.79 \times 10^{-8} \text{ s}}$$

### Solution of Question ③

I.

Call the wire along the  $x$ -axis wire 1 and the other wire 2. Also, choose the positive direction for the magnetic fields at point  $P$  to be out of the page. At point  $P$ ,

$$B_{\text{net}} = +B_1 - B_2 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi} \left( \frac{I_1}{r_1} - \frac{I_2}{r_2} \right)$$

Substituting numerical values,

$$B_{\text{net}} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})}{2\pi} \left( \frac{7.00 \text{ A}}{3.00 \text{ m}} - \frac{6.00 \text{ A}}{4.00 \text{ m}} \right) = +1.67 \times 10^{-7} \text{ T}$$

$$\vec{B}_{\text{net}} = \boxed{0.167 \mu\text{T out of the page}}$$

II.

- (a) The motional emf induced in the bar must be  $\mathcal{E} = IR$ , where  $I$  is the current in this series circuit. Since  $\mathcal{E} = B\ell v$ , the speed of the moving bar must be

$$v = \frac{\mathcal{E}}{B\ell} = \frac{IR}{B\ell} = \frac{(8.50 \times 10^{-3} \text{ A})(9.00 \ \Omega)}{(0.300 \text{ T})(0.350 \text{ m})} = \boxed{0.729 \text{ m/s}}$$

- (b) The flux through the closed loop formed by the rails, the bar, and the resistor is directed into the page and is increasing in magnitude. To oppose this change in flux, the current must flow in a manner so as to produce flux out of the page through the area enclosed by the loop. This means the current will flow counterclockwise.

- (c) The rate at which energy is delivered to the resistor is

$$P = I^2 R = (8.50 \times 10^{-3} \text{ A})^2 (9.00 \ \Omega) \\ = 6.50 \times 10^{-4} \text{ W} = \boxed{0.650 \text{ mW}}$$

- (d) Work is being done by the external force, which is transformed into internal energy in the resistor.

III.

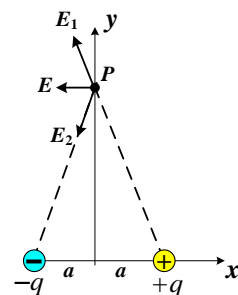
$$|\mathcal{E}| = \left| \frac{\Delta \Phi_B}{\Delta t} \right| = \frac{\Delta(\vec{B} \cdot \vec{A})}{\Delta t} \\ = \frac{(2.50 \text{ T} - 0.500 \text{ T})(8.00 \times 10^{-4} \text{ m}^2)}{1.00 \text{ s}} \left( \frac{1 \text{ N} \cdot \text{s}}{1 \text{ T} \cdot \text{C} \cdot \text{m}} \right) \left( \frac{1 \text{ V} \cdot \text{C}}{1 \text{ N} \cdot \text{m}} \right) \\ = 1.60 \text{ mV}$$

We then find the current induced in the loop from

$$I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{1.60 \text{ mV}}{2.00 \ \Omega} = \boxed{0.800 \text{ mA}}$$

Solution of Question 4

1. [3 marks] An electric dipole is formed by two charges: a charge  $+q$  at  $(a, 0)$  and a charge  $-q$  at  $(-a, 0)$ . The electric field at any point at the  $y$ -axis is in the .....
- (a) positive  $x$ -direction      (b) negative  $y$ -direction      (c) positive  $z$ -direction  
 (d) positive  $y$ -direction      **(e) negative  $x$ -direction**      (f) negative  $z$ -direction



2. [3 marks] A point charge  $Q$  is placed at  $(0, 0)$ . The electric flux over a sphere of radius 2 m and centered at  $(0, 4)$  is .....
- (a)  $Q \cdot \epsilon_0$       (b)  $\epsilon_0/Q$       (c)  $Q/\epsilon_0$       **(d) zero**      (e)  $2Q/\epsilon_0$   
 (f)  $4Q/\epsilon_0$

The charge is located outside the surface, thus,  $Q_{\text{enc}} = 0$  and the flux is zero.

3. [3 marks] If both the plate area and the plate separation of a parallel-plate capacitor are doubled, the capacitance will .....
- (a) increase 4 times      **(b) be the same**      (c) decrease 4 times  
 (d) be doubled      (e) be halved      (f) increase 6 times

$$C = \epsilon A/d, \quad C' = \frac{\epsilon(2A)}{2d} = \frac{\epsilon A}{d} = C$$

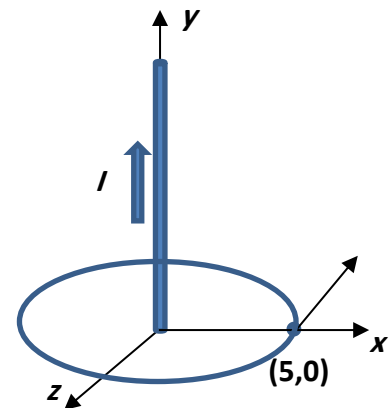
4. [3 marks] If the length of a resistor is doubled and its cross-sectional area is halved, the resistance of the resistor will be .....
- (a) doubled      (b) halved      **(c) four times larger**  
 (d) four times less      (e) the same      (f) eight times larger

$$R' = \frac{\rho l'}{A'} = \frac{\rho 2l}{A/2} = \frac{4\rho l}{A} = 4R$$

5. [3 marks] A particle is moving in a circular path when it moves perpendicularly to a magnetic field  $B$ . The angular speed of this particle ..... the magnetic field  $B$ .
- (a) is inversely proportional to      (b) does not depend on  
**(c) is directly proportional to**      (d) is directly proportional to the square of  
 (e) is inversely proportional to the square of

$$\omega = qB/m$$

6. [3 marks] An infinitely long wire along the  $y$ -axis is carrying a current in the positive  $y$ -direction. The magnetic field at the point  $(5, 0)$  is .....
- (a) positive  $x$ -direction      (b) negative  $y$ -direction  
 (c) positive  $z$ -direction      (d) positive  $y$ -direction  
 (e) negative  $x$ -direction      **(f) negative  $z$ -direction**



7. [3 marks] A square loop of wire lies in the plane of the page. A decreasing magnetic field is directed into the page. The induced current in the loop is.....
- (a) counterclockwise      **(b) clockwise**      (c) zero      (d) out of the page  
 (e) into the page

By lenz's law: since the field is decreasing, then the induced current should produce a magnetic field in the same direction of the original field.

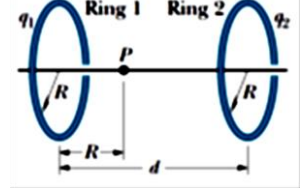
8. [3 marks] The energy stored in coil is  $U$ . If the current in the coil is halved, then the energy stored will become ..... the original value
- (a)  $U/2$       **(b)  $U/4$**       (c)  $U$       (d)  $2U$       (e)  $4U$       (f)  $16U$

$$U' = \frac{1}{2} L I'^2 = \frac{1}{2} L \left(\frac{I}{2}\right)^2 = \frac{U}{4}$$

**Answer All questions (90 marks)**

**Question ①**

22 marks

- I. [7 marks, CLO: a1, b2, c1] The Figure shows two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge  $q_1$  and radius  $R$ ; ring 2 has uniform charge  $q_2$  and the same radius  $R$ . The rings are separated by distance  $d = 4R$ . The net electric field at point  $P$  on the common line, at distance  $R$  from ring 1, is zero. What is the ratio  $q_2/q_1$ ?
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- II. [7 marks, CLO: a1, b2] Charge  $Q$  is uniformly distributed in a sphere of radius  $R$ . (a) what fraction of the charge is contained within the radius  $r = R/3$ ? (b) What is the ratio of the electric field magnitude at  $r = R/2$  to that on the surface of the sphere?
- III. [8 marks, CLO: a1, b2, c1] (a) Points A [at (3, 6) m] and B [at (8, -3) m] are in a region where the electric field is uniform and given by  $\vec{E} = 12\hat{i}$  N/C. What is the electric potential difference  $V_A - V_B$ ? (b) Two charged spherical conductors are connected by a long conducting wire, and a charge of  $20.0 \mu\text{C}$  is placed on the combination. If one sphere has a radius of 4.00 cm and the other has a radius of 6.00 cm, what is the electric potential of each sphere?

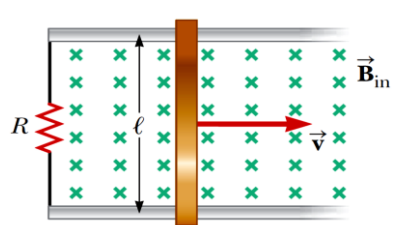
**Question ②**

22 marks

- I. [7 marks, CLO: a3, b2, c1] A 2.00-nF parallel-plate capacitor is charged to an initial potential difference  $V = 100$  V and is then isolated. The dielectric material between the plates is mica, with a dielectric constant of 5.00. What is the capacitance, charge, potential difference, and energy stored in the capacitor after the mica is withdrawn?
- II. [8 marks, CLO: a1, b2, c1] An aluminum wire having a cross-sectional area equal to  $4.00 \times 10^{-6} \text{ m}^2$  carries a current of 5.00 A. The density of aluminum is  $2.70 \text{ g/cm}^3$ . Assume each aluminum atom supplies one conduction electron per atom. Find the drift speed of the electrons in the wire. (Molar mass of aluminum is 27 g/mol, Avogadro's number is  $6.022 \times 10^{23}$ )
- III. [7 marks, CLO: a3, b2] An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 2.00 mT. If the speed of the electron is  $1.50 \times 10^7$  m/s, determine (a) the radius of the circular path and (b) the time interval required to complete one revolution.

**Question ③**

22 points

- I. [7 marks, CLO: a1, b2] A wire carries a 7.00-A current along the +x axis, and another wire carries a 6.00-A current along the +y axis. What is the magnetic field at point P, located at  $x = 4.00$  m,  $y = 3.00$  m?
- II. [8 marks, CLO: a3, b2, c1] A conducting bar of length  $\ell$  moves to the right on two frictionless rails as shown in Figure below. A uniform magnetic field directed into the page has a magnitude of 0.300 T. Assume  $R = 9.00 \Omega$  and  $\ell = 0.350$  m. (a) At what constant speed should the bar move to produce an 8.50-mA current in the resistor? (b) What is the direction of the induced current? (c) At what rate is energy delivered to the resistor? (d) Explain the origin of the energy being delivered to the resistor.
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III. [7 marks, CLO: a1, b2, c1] A flat loop of wire consisting of a single turn of cross-sectional area  $8.00 \text{ cm}^2$  is perpendicular to a magnetic field that increases uniformly in magnitude from  $0.500 \text{ T}$  to  $2.50 \text{ T}$  in  $1.00 \text{ s}$ . What is the resulting induced current if the loop has a resistance of  $2.00 \Omega$ ?

**Question 4**

24 points

**Choose the correct answer justifying your choice (answers without justification are ignored):**

1. [3 marks, CLO: a1, b2] An electric dipole is formed by two charges: a charge  $+q$  at  $(a, 0)$  and a charge  $-q$  at  $(-a, 0)$ . The electric field at any point at the  $y$ -axis is in the .....  
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2. [3 marks, CLO: a1, b2] A point charge  $Q$  is placed at  $(0, 0)$ . The electric flux over a sphere of radius  $2 \text{ m}$  and centered at  $(0, 4)$  is .....  
 (a)  $Q \cdot \epsilon_0$       (b)  $\epsilon_0 / Q$       (c)  $Q / \epsilon_0$       (d) zero      (e)  $2Q / \epsilon_0$       (f)  $4Q / \epsilon_0$

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3. [3 marks, CLO: a3, b2, c1] If both the plate area and the plate separation of a parallel-plate capacitor are doubled, the capacitance will .....  
 (a) increase 4 times      (b) be the same      (c) decrease 4 times  
 (d) be doubled      (e) be halved      (f) increase 6 times

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4. [3 marks, CLO: a1, b2, c1] If the length of a resistor is doubled and its cross-sectional area is halved, the resistance of the resistor will be .....  
 (a) doubled      (b) halved      (c) four times larger  
 (d) four times less      (e) the same      (f) eight times larger

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5. [3 marks, CLO: a3, b2] A particle is moving in a circular path when it moves perpendicularly to a magnetic field  $B$ . The angular speed of this particle .....the magnetic field  $B$ .  
 (a) is inversely proportional to      (b) does not depend on  
 (c) is directly proportional to      (d) is directly proportional to the square of  
 (e) is inversely proportional to the square of

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6. [3 marks, CLO: a1, b2] An infinitely long wire along the  $y$ -axis is carrying a current in the positive  $y$ -direction. The magnetic field at the point  $(5, 0)$  is .....  
 (a) positive  $x$ -direction      (b) negative  $y$ -direction      (c) positive  $z$ -direction  
 (d) positive  $y$ -direction      (e) negative  $x$ -direction      (f) negative  $z$ -direction

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7. [3 marks, CLO: a3, b2, c1] A square loop of wire lies in the plane of the page. A decreasing magnetic field is directed into the page. The induced current in the loop is.....  
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8. [3 marks, CLO: a1, b2, c1] The energy stored in coil is  $U$ . If the current in the coil is halved, then the energy stored will become ..... the original value  
 (a)  $U/2$       (b)  $U/4$       (c)  $U$       (d)  $2U$       (e)  $4U$       (f)  $16U$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

$$q_{e,p} = 1.6 \times 10^{-19} C$$

$$k_e = 9 \times 10^{-9} \frac{N \cdot m^2}{C^2}$$

$$m_e = 9.1 \times 10^{-31} kg$$

$$m_p = 1.67 \times 10^{-27} kg$$

$$G = 6.67 \times 10^{-11} \frac{m^3}{s^2 \cdot kg}$$

*End of the exam*

*Best wishes*