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Answers of Final Written Exam

Answers of Question ① 《20 points》

a) [5 points CLO: a1, a3, b3**]** The average threshold of dark-adapted vision is 4.00 × 10^{-11} W/m² at a central wavelength of 500 nm. If light with this intensity and wavelength enters the eye and the pupil is open to its maximum diameter of 8.50 mm, how many photons per second enter the eye?

The energy of a single 500-nm photon is:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{500\times10^{-9}\ \text{m}}$$
$$= 3.98\times10^{-19}\ \text{J}$$

The energy entering the eye each second

$$E = P\Delta t = IA\Delta t$$
$$= (4.00\times10^{-11}\ \text{W/m}^2)\left[\frac{\pi}{4}(8.50\times10^{-3}\ \text{m})^2\right](1.00\ \text{s})$$
$$= 2.27\times10^{-15}\ \text{J}$$

The number of photons required to yield this energy is

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15}\ \text{J}}{3.98\times10^{-19}\ \text{J/photon}} = \boxed{5.71\times10^{3}\ \text{photons}}$$

b) [5 points CLO: a1, a3, b3**]** A 650-keV gamma ray Compton-scatters from an electron. Find the energy of the photon scattered at 110°, the kinetic energy of the scattered electron, and the recoil angle of the electron.

$$\lambda' = \lambda + \lambda_c(1-\cos\theta) = \frac{hc}{E} + \lambda_c(1-\cos\theta);$$

$$\lambda' = \frac{1240\ \text{eV}\cdot\text{nm}}{650\times10^{3}\ \text{eV}} + (2.43\times10^{-3}\ \text{nm})(1-\cos110^\circ) = 5.17\ \text{pm}$$

$$E' = \frac{hc}{\lambda'} = \frac{1240\ \text{eV}\cdot\text{nm}}{5.17\times10^{-3}\ \text{nm}} = 2.40\times10^{5}\text{eV} = 240\,\text{keV}$$

By conservation of energy we find: $K_e = E - E' = 650\,\text{keV} - 240\,\text{keV} = 410\,\text{keV}$

Conservation of p_x: $p_e\cos\phi + \frac{h}{\lambda'}\cos\theta = \frac{h}{\lambda}$;

$$p_e\cos\phi = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta. \qquad (1)$$

Conservation of p_y: $p_e\sin\phi - \frac{h}{\lambda'}\sin\theta = 0$;

$$p_e\sin\phi = \frac{h}{\lambda'}\sin\theta \qquad (2)$$

Dividing equation (2) by equation (1), $\tan\phi = \dfrac{\frac{h}{\lambda'}\sin\theta}{\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta}$

$$\phi = 17.1^\circ$$

c) [5 points CLO: a1, a3**]** Derive an expression for the quantized energy levels of the hydrogen atom.

The electric potential energy of the system is given by:

$$U = k_e\frac{q_1q_2}{r} = \frac{-k_e e^2}{r},$$

where k_e is the Coulomb constant and the negative sign arises from the charge $-e$ on the electron. Therefore, the *total* energy of the atom, which consists of the electron's kinetic energy and the system's potential energy, is

$$E = K + U = \tfrac{1}{2}m_e v^2 - k_e \frac{e^2}{r}$$

The electron is modeled as a particle in uniform circular motion, so the electric force $k_e e^2/r^2$ exerted on the electron must equal the product of its mass and its centripetal acceleration ($a_c = v^2/r$):

$$\frac{k_e e^2}{r^2} = \frac{m_e v^2}{r}$$

$$v^2 = \frac{k_e e^2}{m_e r}$$

The kinetic energy of the electron is

$$K = \tfrac{1}{2}m_e v^2 = \frac{k_e e^2}{2r}$$

Substituting this value of K into the equation of energy E gives the following expression for the total energy of the atom:

$$E = -\frac{k_e e^2}{2r}$$

Because the total energy is *negative*, which indicates a bound electron–proton system, energy in the amount of $k_e e^2/2r$ must be added to the atom to remove the electron and make the total energy of the system zero.

To obtain an expression for r, the radius of the allowed orbits, solving the equation of the angular momentum for v^2 and equating it to the one obtained from the centripetal force yields:

$$v^2 = \frac{n^2\hbar^2}{m_e^2 r^2} = \frac{k_e e^2}{m_e r}$$

$$r_n = \frac{n^2\hbar^2}{m_e k_e e^2} \qquad n = 1, 2, 3, \ldots$$

The orbit with the smallest radius, called the **Bohr radius** a_o , corresponds to $n = 1$ and has the value

$$a_0 = \frac{\hbar^2}{m_e k_e e^2} = 0.052\,9 \text{ nm}$$

Substituting this equation into that of the radius gives a general expression for the radius of any orbit in the hydrogen atom:

$$r_n = n^2 a_0 = n^2(0.052\,9 \text{ nm}) \qquad n = 1, 2, 3, \ldots$$

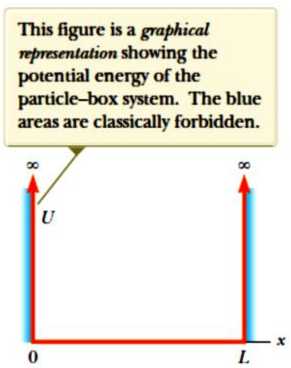
The quantization of orbit radii leads to energy quantization. Substituting $r_n = n^2 a_o$ into the equation of energy gives

$$E_n = -\frac{k_e e^2}{2a_0}\left(\frac{1}{n^2}\right) \qquad n = 1, 2, 3, \ldots$$

Inserting numerical values into this expression, we find that

$$E_n = -\frac{13.606 \text{ eV}}{n^2} \qquad n = 1, 2, 3, \ldots$$

d) [5 points CLO: a1, a3**]** Apply Schrödinger equation and boundary conditions to find the energy levels and wave functions for an electron in one-dimensional potential box.



As an application of the Schrödinger equation, a simple system of a particle trapped in a one-dimensional box is considered with infinitely hard walls that the particle cannot penetrate. The potential, called an *infinite square well*, is shown in Figure 1.12 and is given by

$$U(x) = \begin{cases} \infty & x \le 0,\ x \ge L \\ 0 & 0 < x < L \end{cases}$$

Infinite square-well potential, the potential is $U = \infty$ everywhere except the region $0 < x < L$, where $U = 0$.

Solving the Schrödinger equation in the region of zero potential

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$$

It can be easily found that the general solution for this equation is

$$\psi(x) = A\sin kx + B\cos kx$$

Because the walls are impenetrable, there is zero probability of finding the particle outside the box, so the wave function $\psi(x)$ must be zero for $x < 0$ and $x > L$. To be a mathematically well-behaved function, $\psi(x)$ must be continuous in space. There must be no discontinuous jumps in the value of the wave function at any point. Therefore, if ψ is zero outside the walls, it must also be zero *at* the walls; that is, $\psi(0) = 0$ and $\psi(L) = 0$.

Only those wave functions that satisfy these boundary conditions are allowed.

So, applying the first boundary condition $\psi(0) = 0$:

$$\psi(0) = A\sin 0 + B\cos 0 = 0 + B = 0$$

, which means that $B = 0$. Therefore, our solution reduces to

$$\psi(x) = A\sin kx$$

The second boundary condition, $\psi(L) = 0$, when applied to the reduced solution gives

$$\psi(L) = A\sin kL = 0$$

The boundary condition is satisfied if kL is an integral multiple of π, that is, if $kL = n\pi$, where n is an integer.

$$kL = \frac{\sqrt{2mE}}{\hbar}L = n\pi$$

Each value of the integer n corresponds to a quantized energy that we call E_n. Solving for the allowed energies E_n gives

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2 \quad n = 1, 2, 3, \ldots$$

The lowest allowed energy corresponds to the **ground state,** which is the lowest energy state for any system. For the particle in a box, the ground state corresponds to $n = 1$, for which $E_1 = h^2/8mL^2$. Because $E_n = n^2E_1$, the **excited states** corresponding to $n = 2, 3, 4 \ldots$ have energies given by $4E_1, 9E_1, 16E_1, \ldots$. (Fig. 1.13)

Substituting the values of k in the wave function, the allowed wave functions $\psi_n(x)$ are given by

$$\psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$$

Using the normalization property

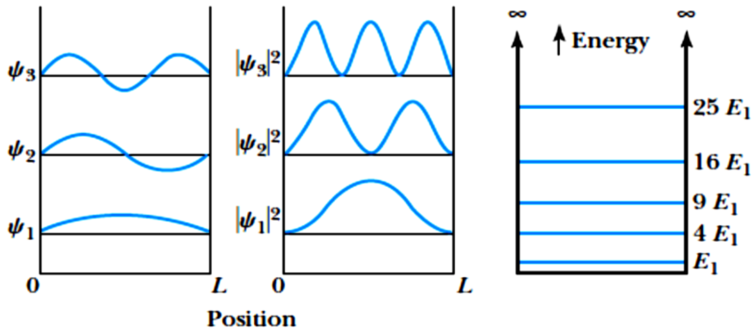
$$\int_{-\infty}^{\infty} |\psi|^2\, dx = 1$$

and Substituting the wave function yields (only integration from 0 to L is considered)

$$A^2\int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

This is a straightforward integral which gives $A^2(L/2) = 1$ and $A = \sqrt{2/L}$. The normalized wave function becomes

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \quad (n = 1, 2, 3, \ldots)$$



Answers of Question ② **《20 points》**

a) [7 points CLO: a1, a3, b3**]** An electron has a kinetic energy of 12.0 eV. The electron is incident upon a rectangular barrier of height 20.0 eV and width 1.00 nm. If the electron absorbed all the energy of a photon of green light (with wavelength 546 nm) at the instant it reached the barrier, by what factor would the electron's probability of tunneling through the barrier increase?

The original tunneling probability is $T = e^{-2CL}$, where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(9.11\times10^{-31}\text{ kg})(20.0-12.0)(1.60\times10^{-19}\text{ J})}}{6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi} = 1.448\ 1\times10^{10}\text{ m}^{-1}$$

The photon energy is $hf = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{546\text{ nm}} = 2.27\text{ eV}$, to make the electron's new kinetic energy $12.0 + 2.27 = 14.27$ eV and its decay coefficient inside the barrier

$$C' = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(9.11\times10^{-31}\text{ kg})(20.0-14.27)(1.60\times10^{-19}\text{ J})}}{6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi} = 1.225\ 5\times10^{10}\text{ m}^{-1}$$

Now the factor of increase in transmission probability is

$$\frac{e^{-2C'L}}{e^{-2CL}} = e^{2L(C-C')} = e^{2(1.00\times10^{-9}\text{ m})(0.223\times10^{10}\text{ m}^{-1})} = e^{4.45} = \boxed{85.9}$$

b) [6 points CLO: a1, a3**] (i)** Write out the electronic configuration of the ground state for molybdenum (Z = 42). **(ii)** Write out the values for the possible set of quantum numbers n, ℓ, m_ℓ, and m_s for the electrons in molybdenum.

(i) $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^5\ 5s^1$ **or** $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^1\ 4d^5$

(ii)

Molybdenum														
n	1	2				3								
ℓ	0	0	1			0	1			2				
m$_\ell$	0	0	–1	0	+1	0	–1	0	+1	–2	–1	0	+1	+2
m$_s$	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓

Molybdenum																			
n	4																5		
ℓ	0	1			2					3							0		
m_ℓ	0	–1	0	+1	–2	–1	0	+1	+2	-3	–2	–1	0	+1	+2	+3	0		
m_s	↑↓	↑↓	↑↓	↑↓		↑↓	↑↓	↑									↑		

c) [7 points CLO: a1, a3, b3**]** A crystal is composed of two elements, A and B. The basic crystal structure is body centered cubic with element A at each of the corners and element B in the center of the cube. The effective radius of element A is r_A = 1.035 Å. Assume that the elements are hard spheres with the surface of each A-type atom in contact with the surface of its nearest A-type neighbor. Calculate **(i)** the maximum radius of the B-type element that will fit into this structure, **(ii)** the lattice constant, and **(iii)** the volume density (#/cm3) of both the A-type atoms and the B-type atoms.

(a)

$$2(r_A)\sqrt{3} = 2(1.035) + 2r_B$$

$$2(1.035)\sqrt{3} = 2(1.035) + 2r_B$$

$$r_B = 0.7576\,\overset{\circ}{\mathrm{A}}$$

(b)

$$a = 2(1.035) = 2.07\,\overset{\circ}{\mathrm{A}}$$

(c)

$$\text{A-atoms: \# of atoms } = 8 \times \frac{1}{8} = 1$$

$$\text{Density } = \frac{1}{\left(2.07\times10^{-8}\right)^3}$$

$$= 1.13\times10^{23}\ \mathrm{cm}^{-3}$$

B-atoms: # of atoms = 1

$$\text{Density } = \frac{1}{\left(2.07\times10^{-8}\right)^3}$$

$$= 1.13\times10^{23}\ \mathrm{cm}^{-3}$$

Answers of Question ③ **❰26 points❱**

1. [2 points] At zero absolute temperature, the conduction band of a conductor ....
(a) is completely empty (b) is completely filled ***(c) is partially filled***
(d) is of lower energy than the valence band (e) none of the above choices
Justification: At 0K for semiconductors and insulators, CB is completely empty but for conductors it is partially filled. CB cannot be completely filled in any case.

2. [2 points] At room temperature, the holes in the valence band of an intrinsic semiconductor ....
(a) are zero ***(b) are equal to the electrons in the conduction band***
(c) do not contribute to current (d) are less than electrons in the conduction band
(e) none of the above choices
Justification: The carrier concentrations are zero at 0K not at room temperature and they contribute to current because the valence band is not completely filled.

3. [2 points] At absolute temperature T, the effective density of valence band states is proportional to
(a) T (b) T^2 (c) T^3 (d) $T^{2/3}$ ***(e) none of the above choices***
Justification: $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$

4. [2 points] The Fermi level of a p-type material ....
(a) increases with temperature (b) is constant with acceptor doping concentration
(c) is constant with temperature (d) increases with acceptor doping concentration
(e) none of the above choices

Justification: For p-type material, $p >> n$, and the Fermi level is near to the bottom of the gap, and increasing temperature will increase n relative to p and lead to make E_F near the middle of the gap.

5. [2 points] Consider a heavily doped p-type silicon sample at equilibrium which has electron and hole concentrations n and p, respectively. If it is heated from T = 200 to 250 K, ...

(a) n and p will increase with the same ratio (b) both n and p will decrease

(c) n will increase but p will decrease (d) both n and p will be nearly the same

(e) n will decrease but p will increase ***(f) none of the above choices***

Justification: T = 200 represents the extrinsic region. For p-type material, $p >> n$, p is nearly constant in the extrinsic region while n increases with temperature.

6. [2 points] Consider an n-type semiconductor sample at equilibrium. If the donor doping is increased in this sample, ...

(a) both mobility and conductivity will increase

(b) mobility will increase but conductivity will decrease

(c) both mobility and conductivity will decrease

(d) mobility will decrease but conductivity will increase

(e) mobility will decrease but conductivity will be the same

Justification: With increasing doping mobility decreases and the electron concentration will increase. Since $\sigma \approx qn\mu_n$, so conductivity will increase.

7. [2 points] If the concentration of electrons increases in the positive x direction, electron diffusion flux will be ....

(a) in the positive x direction ***(b) in the negative x direction***

(c) zero (d) in the positive y direction (d) in the negative y direction

Justification: The flux is from the region of higher concentration to that of lower concentration

8. [2 points] A current passing in the positive x-direction normal to a differential volume element of a semiconductor at steady state. If the generation and recombination rates of electrons are 2×10^{19} cm^{-3}s^{-1} and 7×10^{19} cm^{-3}s^{-1}, respectively, the gradient of electron current density is ....

(a) 5 A/cm^3 (b) –5 A/cm^3 (c) zero (d) –8 A/cm^3 ***(e) 8 A/cm³***

Justification: $\frac{\partial n}{\partial t} = +\frac{1}{q}\frac{\partial J_n}{\partial x} + G - R$

At steady state, $\frac{\partial n}{\partial t} = 0$, $G - R = -5\times10^{19}$ cm^{-3}s^{-1} ➔ $\frac{\partial J_n}{\partial x} = +5\times10^{19}\,(1.6\times10^{-19}) = 8\ A/cm^3$

9. [2 points] For a pn-junction at equilibrium, the electric potential is .... the depletion layer.

(a) maximum at the n-edge of (b) constant within (c) zero within

(d) minimum at the n-edge of (e) increasing linearly within

Justification:

Since electrons diffuse from the n-side to the p-side, positive ions are formed at the n-side and the potential is higher at this side.

10. [2 points] For a pn-junction at equilibrium, the built-in potential is inversely proportional to ...

(a) donor doping density ***(b) temperature*** (c) bandgap energy

(d) acceptor doping density (e) none of the above choices

Justification: $V_{bi} = V_T \ln\frac{N_A N_D}{n_i^2}$ increasing temperature increases n_i and so V_{bi} decreases

11. [2 points] The reverse saturation current of a pn-junction is directly proportional to ......

(a) doping densities (b) bandgap energy of the junction material

(c) the length of the neutral regions (d) minority carrier lifetimes

(e) temperature of the junction (f) none of the above choices

Justification:

$$I_s = qA\left[\frac{D_p p_{no}}{L_p} + \frac{D_n n_{po}}{L_n}\right] = qA\left[\frac{D_p}{L_p}\frac{n_i^2}{N_d} + \frac{D_n}{L_n}\frac{n_i^2}{N_a}\right]$$

increasing temperature increases n_i and so I_s increases

12. [2 points] Plotting the diode current versus diode voltage of an ideal silicon diode on a semi-log scale (log y versus x) in the voltage range 0.8 – 2 V results in ....

(a) an exponential decay curve (b) an exponential rise curve

(c) a straight line of positive slope (d) a straight line of negative slope

(e) a horizontal straight line

Justification:

$$I_D = I_s\left(e^{V_D/V_T} - 1\right) \approx I_s\left(e^{V_D/V_T}\right)$$

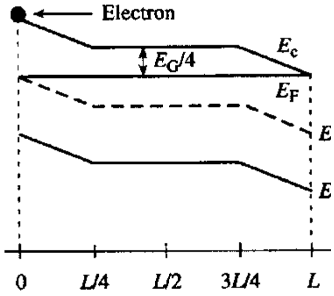
$$\ln I_D = \ln I_s + V_D/V_T$$

13. [2 points] The dynamic resistance of an ideal silicon diode which is reverse biased at a voltage of 10 V is ………………… (reverse saturation current is 25.5 nA).

(a) nearly 1 MΩ (b) ***greater than 100 GΩ*** (c) nearly 1000 MΩ
(d) smaller than 10 Ω (e) nearly 1 μΩ

Justification: $\frac{dI_D}{dV_D} = \frac{1}{V_T} I_s \left(e^{V_D/V_T}\right) \rightarrow r_D = \frac{dV_D}{dI_D} = \frac{V_T}{I_s\left(e^{V_D/V_T}\right)}$ >100 GΩ

Answers of Question ④ **《14 points》**



With regard to the energy band diagram of a silicon sample that is shown beside, answer the following questions given that L = 1 μm:

1. **[2 points** CLO: a1, a3**]** Specify all the values of x for which the sample is of p-type.
Answer: no where as E_F is always > E_i

2. **[2 points** CLO: a1, a3**]** Specify the point at which the electric potential is maximum.
Answer: potential is maximum when Ec is minimum (x = L)

3. **[2 points** CLO: a1, a3, b3**]** Find the hole concentration at x = 0.
Answer: p = n = n_i = 1×10^{10} cm^{-3}.

4. **[2 points** CLO: a1, a3, b3**]** Find the electron concentration at $x = L/2$.
Answer: n = n_i exp(E_f - E_{fi})/kT = 1×10^{10} **exp ((1.12/4)/0.0258) =**

5. **[2 points** CLO: a1, a3, b3**]** Find the electric field at $x = L/8$.
Answer: $E = \frac{1}{q}\frac{dEc}{dx} = \frac{1}{q}(\frac{EG}{4})/(L/4)$ = E_G/L = 1.12/1 = 1.12 V/μm

6. **[2 points** CLO: a1, a3, b3**]** Find the kinetic energy of the electron shown in the diagram.
Answer: *K.E.* = 0

7. **[1 point** CLO: a1, a3, b3**]** Find the direction of the hole drift current at $x = 7L/8$.
Answer: The drift current is in the same direction of the electric field which is negative x direction

8. **[1 point** CLO: a1, a3, b3**]** Find the magnitude of the electron diffusion current density at $x = L/2$.
Answer: All currents are zero at x = $L/2$ since no electric field and no change in carrier concentrations

Question ⑤ **《10 points》**

A silicon pn-junction of doping densites 5×10^{17} cm^{-3} and 2×10^{16} cm^{-3} at the p-side and n-side, respectivly. The junction is forward-biased by applying 0.5 V on its terminals. The minority carrier lifetime of electrons and holes are 2×10^{-8} and 1×10^{-8} s, respectively.

(i) **[2 points** CLO: a1, a3, b3**]** Find the built in voltage at equilibrium.

Answer: $V_{bi} = V_T ln\frac{N_A N_D}{n_i^2} = 0.0258\ln\left(\frac{5\text{x}10^{17} 2\text{x}10^{16}}{(1\text{x}10^{10})^2}\right) = 0.8334\, V$

(ii) **[2 points** CLO: a1, a3, b3**]** Find the equilibrium carrier concentrations on both sides of the junction.
Answer: p_{po} = 5×10^{17} cm^{-3} , n_{no} = 2×10^{16} cm^{-3} ,
n_{po} = $(1\times10^{10})^2/5\times10^{17}$ = 200 cm^{-3} ,
p_{no} = $(1\times10^{10})^2/2\times10^{16}$ = 5000 cm^{-3}

(iii) **[2 points** CLO: a1, a3, b3**]** Find the electron concentration in the p-side at a distance 8.25 μm from the edge of the depletion layer.
Answer: **D_n** = μ_n V_T = (1500)(0.0258) = 38.77 cm^2/s. $L_n = \sqrt{D_n\tau_n} = \sqrt{(38.77)(2\times10^{-8})} = 8.8\,\mu m$

$$\Delta n_p(x) = \Delta n_p(-x_p)e^{\frac{x+x_p}{L_n}} = n_{po}\left(e^{\frac{V}{V_T}} - 1\right)e^{\frac{x+x_p}{L_n}}$$

$$\Delta n_p(x) = 200\left(e^{\frac{0.5}{0.0258}} - 1\right)e^{\frac{8.25}{8.8}} = 1.3\times10^{11} cm^{-3}$$

$$n_p = n_{po} + \Delta n_p = 200 + 1.3\times10^{11} \approx 1.3\times10^{11}\ cm^{-3}$$

(iv) **[2 points** CLO: a1, a3, b3**]** Find the electron current density in the p-side at a distance 8.25 μm from the edge of the depletion layer.
Answer:

$$J_n(x) = qD_n\frac{dn_p}{dx} = \frac{qD_n n_{po}}{L_n}\left(e^{\frac{V}{V_T}} - 1\right)e^{\frac{x+x_p}{L_n}}$$

$$J_n(x) = \frac{(1.6\times10^{-19})(38.7)(200)}{8.8\times10^{-4}}\left(e^{\frac{0.5}{0.0258}} - 1\right)e^{\frac{8.25}{8.8}} = 9\times10^{-4}A/cm^2$$

(v) **[2 points** CLO: a1, a3, b3**]** If the junction is illuminated uniformly by light resulting in a uniform generation rate of 5×10^{19} cm^{-3}s^{-1}, what will be the electron concentration in the p-side at a distance 8.25 μm from the edge of the depletion layer?
Answer: The light causes at steady state an excess electron concentration $\Delta n_p(light) = G\tau_n$

$$n_p = 1.3\times10^{11} + 5\times10^{19}(2\times10^{-8}) = 11.3\times10^{11}\ cm^{-3}$$

End of the exam **Examiners:** Prof Tarek M. Abdolkader, Dr Ibrahim Maged *Best wishes*

Benha University
Benha Faculty of Engineering
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Examiners: Physics Staff
Total marks: 90

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Program: Bachelor
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Final Written Exam

CONSTANTS: q = 1.6 × 10^{-19} C, h = 6.626 × 10^{-34} J.s, m_e = 9.1 × 10^{-31} kg, **kT/q at 300 K** = 0.02586 V
k_e = 9×10^9 N.m²/C², m_p = 1.67× 10^{-27} kg, c = 3×10^8 m/s, σ = 5.67×10^{-8} W/m².K⁴, a_0 = 0.0529 nm, R_H = 1.1×10^7 m^{-1}
PROPERTIES OF SILICON: at T = 300 K: E_g = 1.12 eV, n_i = 1 × 10^{10} cm^{-3}, μ_n = 1500 cm²/(V.s), μ_p = 800 cm²/(V.s)

Answer all questions:

Question ① 《20 points》

a) [5 points CLO: a1, a3, b3**]** The average threshold of dark-adapted vision is 4.00 × 10^{-11} W/m² at a central wavelength of 500 nm. If light with this intensity and wavelength enters the eye and the pupil is open to its maximum diameter of 8.50 mm, how many photons per second enter the eye?

b) [5 points CLO: a1, a3, b3**]** A 650-keV gamma ray Compton-scatters from an electron. Find the energy of the photon scattered at 110°, the kinetic energy of the scattered electron, and the recoil angle of the electron.

c) [5 points CLO: a1, a3**]** Derive an expression for the quantized energy levels of the hydrogen atom.

d) [5 points CLO: a1, a3**]** Apply Schrödinger equation and boundary conditions to find the energy levels and wave functions for an electron in one-dimensional potential box.

Question ② 《20 points》

a) [7 points CLO: a1, a3, b3**]** An electron has a kinetic energy of 12.0 eV. The electron is incident upon a rectangular barrier of height 20.0 eV and width 1.00 nm. If the electron absorbed all the energy of a photon of green light (with wavelength 546 nm) at the instant it reached the barrier, by what factor would the electron's probability of tunneling through the barrier increase?

b) [6 points CLO: a1, a3**] (i)** Write out the electronic configuration of the ground state for molybdenum (Z = 42). **(ii)** Write out the values for the possible set of quantum numbers n, ℓ, m_ℓ, and m_s for the electrons in molybdenum.

c) [7 points CLO: a1, a3, b3**]** A crystal is composed of two elements, A and B. The basic crystal structure is body centered cubic with element A at each of the corners and element B in the center of the cube. The effective radius of element A is r_A = 1.035 Å. Assume that the elements are hard spheres with the surface of each A-type atom in contact with the surface of its nearest A-type neighbor. Calculate **(i)** the maximum radius of the B-type element that will fit into this structure, **(ii)** the lattice constant, and **(iii)** the volume density (#/cm3) of both the A-type atoms and the B-type atoms.

Question ③ 《26 points》

Choose the correct answer justifing your choice (answers without justification are ignored):

1. [2 points CLO: a1, a3**]** At zero absolute temperature, the conduction band of a conductor ……….
(a) is completely empty (b) is completely filled (c) is partially filled
(d) is of lower energy than the valence band (e) none of the above choices

2. [2 points CLO: a1, a3**]** At room temperature, the holes in the valence band of an intrinsic semiconductor….
(a) are zero (b) are equal to the electrons in the conduction band
(c) do not contribute to current (d) are less than electrons in the conduction band
(e) none of the above choices

3. [2 points CLO: a1, a3**]** At absolute temperature T, the effective density of valence band states is proportional to …….
(a) T (b) T^2 (c) T^3 (d) $T^{2/3}$ (e) none of the above choices

4. [2 points CLO: a1, a3**]** The Fermi level of a p-type material ………………
(a) increases with temperature (b) is constant with acceptor doping concentration
(c) is constant with temperature (d) increases with acceptor doping concentration
(e) none of the above choices

5. [2 points CLO: a1, a3, b3**]** Consider a heavily doped p-type silicon sample at equilibrium which has electron and hole concentrations n and p, respectively. If it is heated from T = 200 to 250 K, ……….
(a) n and p will increase with the same ratio (b) both n and p will decrease
(c) n will increase but p will decrease (d) both n and p will be nearly the same
(e) n will decrease but p will increase (f) none of the above choices

Continue with the remaining questions at the back

6. **[2 points** CLO: a1, a3**]** Consider an n-type semiconductor sample at equilibrium. If the donor doping is increased in this sample, ...
(a) both mobility and conductivity will increase
(b) mobility will increase but conductivity will decrease
(c) both mobility and conductivity will decrease
(d) mobility will decrease but conductivity will increase
(e) mobility will decrease but conductivity will be the same

7. **[2 points** CLO: a1, a3**]** If the concentration of electrons increases in the positive x direction, electron diffusion flux will be............
(a) in the positive x direction (b) in the negative x direction
(c) zero (d) in the positive y direction (e) in the negative y direction

8. **[2 points** CLO: a1, a3, b3**]** A current passing in the positive x-direction normal to a differential volume element of a semiconductor at steady state. If the generation and recombination rates of electrons are 2×10^{19} cm^{-3}s^{-1} and 7×10^{19} cm^{-3}s^{-1}, respectively, the gradient of electron current density is.........
(a) 5 A/cm^3 (b) –5 A/cm^3 (c) zero (d) –8 A/cm^3 (e) 8 A/cm^3

9. **[2 points** CLO: a1, a3**]** For a pn-junction at equilibrium, the electric potential is ....the depletion layer.
(a) maximum at the n-edge of (b) constant within (c) zero within
(d) minimum at the n-edge of (e) increasing linearly within

10. **[2 points** CLO: a1, a3**]** For a pn-junction at equilibrium, the built-in potential is inversely proportional to
(a) donor doping density (b) temperature (c) bandgap energy
(d) acceptor doping density (e) none of the above choices

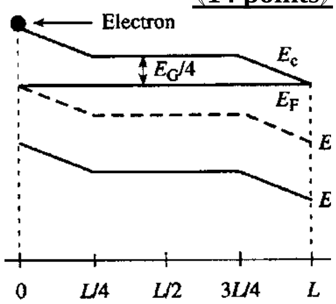
11. **[2 points** CLO: a1, a3**]** The reverse saturation current of a pn-junction is directly proportional to ...
(a) doping densities (b) bandgap energy of the junction material
(c) the length of the neutral regions (d) minority carrier lifetimes
(e) temperature of the junction (f) none of the above choices

12. **[2 points** CLO: a1, a3, b3**]** Plotting the diode current versus diode voltage of an ideal silicon diode on a semi-log scale (log y versus x) in the voltage range 0.8 – 2 V results in ....
(a) an exponential decay curve (b) an exponential rise curve
(c) a straight line of positive slope (d) a straight line of negative slope
(e) a horizontal straight line

13. **[2 points** CLO: a1, a3, b3**]** The dynamic resistance of an ideal silicon diode which is reverse biased at a voltage of 10 V is .... (reverse saturation current is 25.5 nA).
(a) nearly 1 MΩ (b) greater than 100 GΩ (c) nearly 1000 MΩ
(d) smaller than 10 Ω (e) nearly 1 μΩ

Question ④ **《14 points》**

With regard to the energy band diagram of a silicon sample that is shown beside, answer the following questions given that L = 1 μm:



1. **[2 points** CLO: a1, a3**]** Specify all the values of x for which the sample is of p-type.
2. **[2 points** CLO: a1, a3**]** Specify the point at which the electric potential is maximum.
3. **[2 points** CLO: a1, a3, b3**]** Find the hole concentration at $x = 0$.
4. **[2 points** CLO: a1, a3, b3**]** Find the electron concentration at $x = L/2$.
5. **[2 points** CLO: a1, a3, b3**]** Find the electric field at $x = L/8$.
6. **[2 points** CLO: a1, a3, b3**]** Find the kinetic energy of the electron shown in the diagram.
7. **[1 point** CLO: a1, a3, b3**]** Find the direction of the hole drift current at $x = 7L/8$.
8. **[1 point** CLO: a1, a3, b3**]** Find the magnitude of the electron diffusion current density at $x = L/2$.

Question ⑤ **《10 points》**

A silicon pn-junction of doping densites 5×10^{17} cm^{-3} and 2×10^{16} cm^{-3} at the p-side and n-side, respectivly. The junction is forward-biased by applying 0.5 V on its terminal. The minority carrier lifetime of electrons and holes are 2×10^{-8} and 1×10^{-8} s, respectively.

(i) **[2 points** CLO: a1, a3, b3**]** Find the built in voltage at equilibrium.
(ii) **[2 points** CLO: a1, a3, b3**]** Find the equilibrium carrier concentrations on both sides of the junction.
(iii) **[2 points** CLO: a1, a3, b3**]** Find the electron concentration in the p-side at a distance 8.25 μm from the edge of the deplpetion layer.
(iv) **[2 points** CLO: a1, a3, b3**]** Find the electron current density in the p-side at a distance 8.25 μm from the edge of the depletion layer.
(v) **[2 points** CLO: a1, a3, b3**]** If the junction is illuminated uniformly by light resulting in a uniform generation rate of 5×10^{19} cm^{-3}s^{-1}, what will be the electron concentration in the p-side at a distance 8.25 μm from the edge of the depletion layer?

End of the exam **Examiners:** Prof Tarek M. Abdolkader, Dr Ibrahim Maged *Best wishes*